Lösning till tentamen i Reglerteknik 2014-10-28

BL 2014-11-06

1. a) $V(s) = \frac{1}{Js} U(s)$ $\qquad L(s) = \frac{K_p}{Js}$

$$G_{ry}(s) = \frac{L(s)}{1+L(s)} = \frac{K_p}{Js+K_p} = \frac{1}{s+K_p/J} = \frac{1}{s+1} \Rightarrow \underline{\underline{K_p = J}}$$

b) $Y(s) = \frac{1}{(s+1)}\frac{1}{s}$ $\qquad y(t) = 1-e^{-t}$

$$y(t_{5\%}) = 1 - e^{-t_{5\%}} = 0{,}95 \qquad t_{5\%} = \ln 20 = \underline{\underline{3.00}}$$

c)
$$G_{ry}(s) = \frac{J}{Js(1+0.2s/\omega_n+(s/\omega_n)^2)+J} = \frac{\omega_n^2}{s^3+0{,}2\omega_n s^2+\omega_n^2 s+\omega_n^2}$$

Routh-Hurwitz tablå

s^3	1	ω_n^2
s^2	$0.2\omega_n$	ω_n^2
s^1	$\frac{0{,}2\omega_n^3-\omega_n^2}{0.2\omega_n}$	0
s^0	ω_n^2	

stabilt för $\begin{cases} \omega_n > 0 \\ \omega_n^2 - 5\omega_n > 0 \end{cases}$

$\Rightarrow \underline{\underline{\omega_n > 5}}$

d) Robust stabilitet då $|T(j\omega)| < \frac{1}{|\Delta_G(j\omega)|}$

För höga frekvenser gäller allmänt att $|T| << 1$, vilket innebär att en resonans i detta frekvensområde ej ger stabilitetsproblem.

2 a)
$$Y(s) = \frac{1-Ts}{(1+s)(1+2s)} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{1+2s} = \frac{(As+A)(1+2s) + Bs(1+2s) + Cs(s+1)}{s(s+1)(1+2s)} = \frac{(2A+2B+C)s^2+(3A+B+C)s+A}{s(s+1)(1+2s)}$$

$A = 1 \qquad 3+B+C = -T \qquad 2+2B+C = 0$

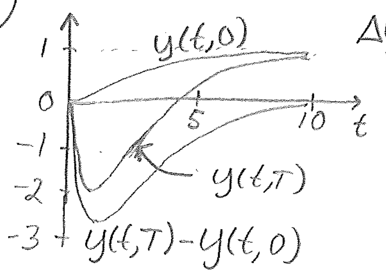
$C = -2-2B \qquad B = -T-3-C = -T-3+2+2B$

$B = T+1 \qquad C = -2-2(T+1) = -4-2T$

$$\underline{\underline{y(t,T) = 1 + (T+1)e^{-t} - (2+T)e^{-0.5t}}}$$

c) Då nollstället $s = \frac{1}{T} \to 0$ går $T \to \infty$
Då $T \to \infty$ går $\min \Delta y(t)$ och $\min y(t,T)$ mot $-\infty$

b)



$$\Delta y(t) = y(t,T) - y(t,0) = Te^{-t} - Te^{-0.5t} = -Te^{-0.5t}\underbrace{(1-e^{-0.5t})}_{\alpha(t)}$$

$\alpha(0) = \alpha(\infty) = 0 \qquad 0 \le \alpha(t) < 1$

$\max \Delta y(t) = 0$

$\min \Delta y(t) = -T \max \alpha(t) \to -\infty$ då $T \to \infty$

3 a) $L_1(s) = \frac{1}{1+2s}\frac{K_{i1}(1+T_{i1}s)}{s}$ $\quad \underline{T_{i1}=2} \Rightarrow L_1(s) = \frac{K_{i1}}{s}$

$$\frac{L_1(s)}{1+L_1(s)} = \frac{K_{i1}}{s+K_{i1}}$$ $\underline{K_{i1}=1}$ ger pol i $s=-1$.

$$L(s) = G_2(s)\frac{L_1(s)}{1+L_1(s)}F_2(s) = \frac{e^{-s}}{1+5s}\frac{1}{1+s}\frac{K_i(1+T_is)}{s}$$

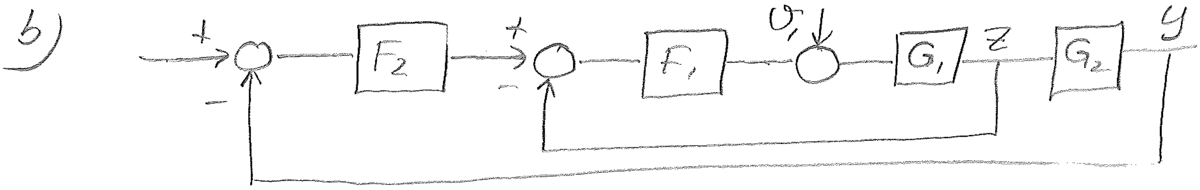
$$\underline{\angle L(j\omega_c)} = -\frac{\omega_c 180^\circ}{\pi} - \arctan(5\omega_c) - \arctan\omega_c - 90^\circ + \arctan(T_i\omega_c)$$

$(\omega_c = 0.4)$ $\quad = -22.9^\circ - 63.4^\circ - 21.8^\circ - 90^\circ + \arctan(T_i\omega_c) = \underbrace{-180^\circ + \varphi_m}_{-135^\circ}$

$$T_i = \frac{1}{0.4}\tan 63.2^\circ = \underline{4.94}$$

$$|L(j\omega_c)| = \frac{K_i\sqrt{1+(T_i\omega_c)^2}}{\sqrt{1+(5\omega_c)^2}\sqrt{1+\omega_c^2}\,\omega_c} = \frac{K_i}{0.435} = 1 \Rightarrow \underline{K_i = 0.435}$$

b)



$$Z = G_1(V_1 + F_1(-F_2Y - Z)) = G_1V_1 - G_1F_1F_2G_2Z - G_1F_1Z$$

$$(1 + G_1F_1(1+G_2F_2))Z = G_1V_1$$

$$Y = G_2Z = \frac{G_1G_2}{1+\underbrace{G_1F_1}_{L_1}(1+G_2F_2)}V_1 =$$

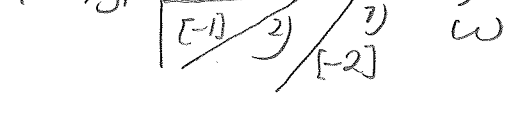
$$G_{v_1y}(s) = \frac{\frac{e^{-s}}{(1+2s)(1+5s)}}{1+\frac{1}{s}\left(1+\frac{e^{-s}K_i(1+T_is)}{s(1+5s)}\right)} \approx \frac{s^2}{K_i}$$ för små s

Lågfrekvensasymptot: $|G_{v_1y}(j\omega)| \approx \frac{\omega^2}{0.435} = \left(\frac{\omega}{0.66}\right)^2$

c) För små s gäller $G(s) \approx 1$ $\quad F(s) \approx \frac{0.32}{s}$

$$G_{v_1y}(s) = \frac{G(s)}{1+G(s)F(s)} \approx \frac{1}{F(s)}$$ för små s

Lågfrekvensasymptot: $|G_{v_1y}(j\omega)| \approx \frac{\omega}{0.32}$



1) Kaskad tydligt effektivare dämpning av v_1 jämfört med
2) Enkel återkoppling.

4. a) $U(z) = \left(K_p + K_i \frac{z^{-1}}{1-z^{-1}}\right) E(z) = \frac{K_p + (K_i - K_p) z^{-1}}{1-z^{-1}} E(z)$

$U(z) = z^{-1} U(z) + K_p E(z) + (K_i - K_p) z^{-1} E(z)$

$u(kh) = u(kh-h) + K_p e(kh) + (K_i - K_p) e(kh-h) =$

$= \underbrace{[u(kh-h) \;\; e(kh) \;\; e(kh-h)]}_{\varphi^T(kh)} \begin{bmatrix} 1 \\ K_p \\ K_i - K_p \end{bmatrix}$

b)
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input(e)
φ(2) = e;
u = φ^T * [1; Kp; Ki-Kp];
output(u)
φ = [u; φ(1:2)];
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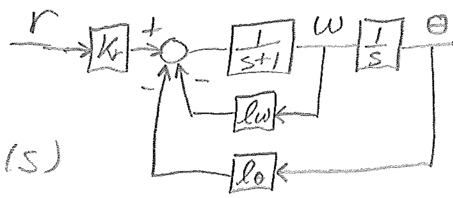
5. a) $s^2 \Theta(s) + s\Theta(s) = U(s)$



$U(s) = -l_\theta \Theta(s) - l_\omega s \Theta(s) + K_r R(s)$

$(s^2 + (1 + l_\omega) s + l_\theta) \Theta(s) = K_r R(s)$

$G_{r\theta}(s) = \frac{K_r}{s^2 + (1+l_\omega) s + l_\theta} = \frac{K_r}{(s+\alpha)^2} = \frac{K_r}{s^2 + 2\alpha s + \alpha^2}$

$\Rightarrow l_\omega = 2\alpha - 1 \quad l_\theta = \alpha^2 \qquad G_{r\theta}(0) = \frac{K_r}{\alpha^2} = 1 \Rightarrow K_r = \alpha^2$

b) $U(s) = -(\alpha^2 + (2\alpha-1)s)\Theta(s) + K_r R(s) = (-(\alpha^2 + (2\alpha-1)s) G_{r\theta}(s) + K_r) R(s)$

$G_{ru}(s) = -\frac{(\alpha^2 + (2\alpha-1)s)\alpha^2}{s^2 + 2\alpha s + \alpha^2} + \alpha^2 \rightarrow \alpha^2$ då $s \to \infty$

$u(0) = \lim_{s \to \infty} s \, G_{ru}(s) \frac{r_0}{s} = \alpha^2 r_0$

↖ ref steg

c) $L(s) = \frac{Z(s)}{U(s)} = (l_\theta + s l_\omega) \frac{\Theta(s)}{U(s)} = \frac{\alpha^2 + (2\alpha-1)s}{s^2 + s}$

$\alpha = 1 \quad L(s) = \frac{s+1}{s(s+1)} = \frac{1}{s} \quad \Rightarrow \quad L(j\omega) = -j\frac{1}{\omega}$

$\angle L(j\omega) = -90^\circ \; \forall \omega \; : \; \varphi_m = 90^\circ$

$\omega_c = 1$ rad/s

$\alpha = 2 \quad L(s) = \frac{4 + 3s}{s(s+1)} = \frac{4(1 + s/1.33)}{s(1+s)}$

Bodediagram $\Rightarrow \varphi_m = 84.6^\circ$

$\omega_c = 3.1$ rad/s