Signaler och System, Z2
SSY042, 2018-06-07

(1)

a)

$H_1(j\omega) = \frac{j\omega}{j\omega + 100}$; $|H_1(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + 100^2}} \to 1$ då $\omega \to \infty$

Högpass

$H_2(j\omega) = \frac{j10\omega}{j\omega + 100}$; $|H_2(j\omega)| = \frac{10\omega}{\sqrt{\omega^2 + 100^2}} \to 10$ då $\omega \to \infty$

Högpass

$H_3(j\omega) = \frac{100}{j\omega + 100} = \frac{1}{1 + j\frac{\omega}{100}}$; $|H_3(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{100}\right)^2}}$

$\to 0$ då $\omega \to \infty$ Lågpass $\omega_{3dB} = 100$

$H_4(j\omega) = \frac{50}{j\omega + 50} = \frac{1}{1 + j\frac{\omega}{50}}$; $|H_4(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{50}\right)^2}}$

$\to 0$ då $\omega \to \infty$ Lågpass $\omega_{3dB} = 50$

Svar: $H_1 - A$ $\quad H_3 - C$
$H_2 - D$ $\quad H_4 - B$

b)

$$x(t) = \underbrace{\pi \cos\left(\frac{3\pi}{11} t\right)}_{x_1(t)} + \underbrace{4e^{j\left(\frac{4}{\pi}\right)t}}_{x_2(t)}, \quad \forall t$$

$x_1(t)$: $\omega_1 = \frac{3\pi}{11}$, $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{3\pi} \cdot 11 = \frac{22}{3}$ s

$x_2(t)$: $\omega_2 = \frac{4}{\pi}$; $T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{4} \cdot \pi = \frac{\pi^2}{2}$ s

Gemensam period $T = k_1 T_1 = k_2 T_2$ med $k_1, k_2 \in \mathbb{Z}$

$\frac{k_1}{k_2} = \frac{T_2}{T_1} = \frac{\pi^2}{2} \cdot \frac{3}{22}$ ej kvot mellan heltal $\Rightarrow$ Icke periodisk

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(2)
$$H(s) = \frac{250}{s^2 + s10 + 125} = \{\text{Kontroll visar komplexa poler}\} =$$

$$= \frac{250}{(s+5)^2 - 25 + 125} = \frac{250}{(s+5)^2 + 10^2} =$$

$$= 25 \cdot \frac{10}{(s+5)^2 + 10^2}$$

Impulssvar:
$$h(t) = \mathcal{L}^{-1}\{H(s)\} = 25e^{-5t}\sin(10t)\,u(t)$$

Stegsvar: Insignal $x(t) = u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s}$

$$Y(s) = X(s) \cdot H(s) = \frac{1}{s} \cdot \frac{250}{(s^2 + s10 + 125)} = \{P.B.U\} =$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + s10 + 125}$$

$$250 = A(s^2 + s10 + 125) + s(Bs + C)$$

$$\left.\begin{array}{ll} s^2: & 0 = A + B \\ s^1: & 0 = 10A + C \\ s^0: & 250 = A \cdot 125 \end{array}\right\} \underline{A = 2} \Rightarrow \underline{B = -2} \Rightarrow \underline{C} = -10A = \underline{-20}$$

$$Y(s) = \frac{2}{s} + \frac{(-2)s - 20}{s^2 + s10 + 125} = 2\left(\frac{1}{s} - \frac{s + 10}{(s+5)^2 + 10^2}\right) =$$

$$= 2\left(\frac{1}{s} - \frac{s + 5 + 5}{(s+5)^2 + 10^2}\right) = 2\left(\frac{1}{s} - \frac{s+5}{(s+5)^2 + 10^2} - \frac{1}{2}\frac{10}{(s+5)^2 + 10^2}\right)$$

Stegsvar $y(t) = \mathcal{L}^{-1}\{Y(s)\} = 2\left(1 - e^{-5t}\left(\cos(10t) + 0{,}5\sin(10t)\right)\right)u(t)$

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(3.) $y[n] - 0{,}2\,y[n-1] - 0{,}24\,y[n-2] = x[n] + 3{,}4\,x[n-1]$

z-transformera

$$Y(z)\left(1 - 0{,}2z^{-1} - 0{,}24z^{-2}\right) = X(z)\left(1 + 3{,}4z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3{,}4z^{-1}}{1 - 0{,}2z^{-1} - 0{,}24z^{-2}} = z \cdot \frac{z + 3{,}4}{z^2 - 0{,}2z - 0{,}24}$$

Poler: $z_{1,2} = 0{,}1 \pm \sqrt{0{,}1^2 + 0{,}24} = 0{,}1 \pm 0{,}5 = \begin{cases} 0{,}6 \\ -0{,}4 \end{cases}$

$$H(z) = z \underbrace{\frac{z + 3{,}4}{(z+0{,}4)(z-0{,}6)}}_{\text{PBU}}$$

$$\frac{z + 3{,}4}{(z+0{,}4)(z-0{,}6)} = \frac{A}{z+0{,}4} + \frac{B}{z-0{,}6}$$

$$z + 3{,}4 = A(z-0.6) + B(z+0{,}4)$$

$z = 0.6 \Rightarrow 0{,}6 + 3{,}4 = B(0{,}6+0{,}4) \Rightarrow B = 4$

$z = -0{,}4 \Rightarrow -0{,}4 + 3{,}4 = A(-0{,}4-0{,}6) \Rightarrow A = -3$

$$H(z) = 4\frac{z}{z-0{,}6} - 3 \cdot \frac{z}{z+0{,}4}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \left[4 \cdot (0{,}6)^n - 3(-0{,}4)^n\right] u[n]$$

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(4) $X[k] = \text{DFT}\{x[n]\}$ $k = 0, 1, 2, \ldots, N-1$

$k = N$ motsvarar "samplingsfrekvensen".

Varje värde på $X[k]$ svarar mot en frekvens i Hz som är $f_k = \frac{k}{N} \cdot f_s$ där f_s är samplingsfrekvensen i Hz.

Vi har $N = 2^{11} = 2048$

$$f_s = \frac{1}{T_s} = \frac{1}{0{,}5 \cdot 10^{-3}} = 2000 \text{ Hz}$$

a/ Frekvens omräknat i index $k = N \cdot \frac{f_k}{f_s}$

Ton	frekv [Hz]	$N \cdot \frac{f_k}{f_s}$	$\lvert X[k]\rvert$ max vid
E	330	$2048 \cdot \frac{330}{2000} = 337{,}9$	$k = 338$ (och $N-k = 1710$)
H	247	$2048 \cdot \frac{247}{2000} = 252{,}9$	$k = 253$ (och $N-k = 1795$)
G	196	$2048 \cdot \frac{196}{2000} = 200{,}7$	$k = 201$ (och $N-k = 1847$)

Notera $\frac{f_s}{2} = 1000 \text{ Hz} < 330 \Rightarrow$ Ingen aliasing (för grundtonen)

b/ Frekvensupplösning

$$\Delta f = \frac{f_s}{N} = \frac{2000}{2048} \approx 0{,}977 \text{ Hz}$$

(5) $x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) =$

$$= \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin\left(n \frac{\pi}{L} t\right)$$

$\omega_0 = \frac{\pi}{L} = \frac{2\pi}{2L} = \frac{2\pi}{T}$; T: Fundamental periodtid

$A_n = 0 \ \forall n$, $B_n = \frac{2}{\pi} \cdot \frac{(-1)^{n+1}}{n}$

$B_1 = \frac{2}{\pi} \cdot 1$; $B_2 = -\frac{2}{\pi} \cdot \frac{1}{2}$; $B_3 = \frac{2}{\pi} \cdot \frac{1}{3}$

Frekvenssvar $G(j\omega) = G(s)\Big|_{s=j\omega} = \frac{j50 \cdot \omega}{40000 - \omega^2 + j50\omega}$

$$|G(j\omega)| = \frac{50\omega}{\sqrt{(40000-\omega^2)^2 + (50\omega)^2}}$$

Beräknas för $\omega = \omega_0$, $2\omega_0$ och $3\omega_0$; $\omega_0 = \frac{\pi}{\pi \cdot 10^{-2}} = 100$ r/s

Amplitud hos de tre lägsta sinusformade sign. i y(t)

$n=1 \quad B_1^Y = |B_1| \cdot |G(j\omega)|\Big|_{\omega=\omega_0} = \frac{2}{\pi} \cdot 0{,}164 \approx 0{,}105$

$n=2 \quad B_2^Y = |B_2| \cdot |G(j\omega)|\Big|_{\omega=2\omega_0} = \frac{1}{\pi} \cdot 1 \approx 0{,}318$

$n=3 \quad B_3^Y = |B_3| \cdot |G(j\omega)|\Big|_{\omega=3\omega_0} = \frac{2}{3\pi} \cdot 0{,}287 \approx 0{,}061$