Signaler och System, Z2
SSY042 170823

1a)

$y[n] = x[n] + \sin(x[n-1])$

- Linjärt?

Insignal | Utsignal

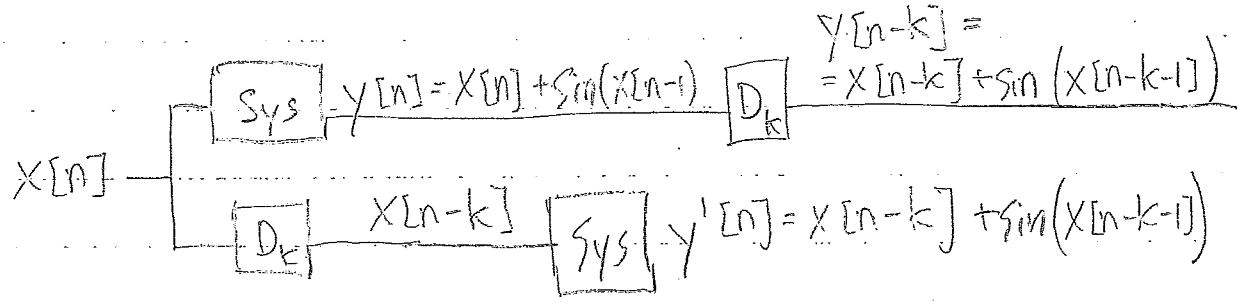
$x[n]$ → $y[n] = x[n] + \sin(x[n-1])$

$x_1[n] = \alpha x[n]$ → $y_1[n] = x_1[n] + \sin(x_1[n-1]) = \alpha x[n] + \sin(\alpha x[n-1])$

$y_1[n] \neq \alpha y[n]$

Ej linjärt

- Tidsinvariant?



$y[n-k] = y'[n]$ Tidsinvariant!

- Kausalt? Ja!

$y[n]$ beror endast av $x[n-k]$ där $k \geq 0$.

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1b)
$$x(t) = \sin(t)\cos(t)\cdot u(t)$$
Trigonometriska samband ger ("tabell")
$$x(t) = \frac{1}{2}\sin(2t)\,u(t)$$
$$X(s) = \mathcal{L}\{x(t)\} = \frac{1}{2}\,\frac{2}{s^2+2^2} = \frac{1}{s^2+4}$$

1c)
$$\mathcal{Z}\{x[n]\} = X(z)$$
$$\mathcal{Z}\{x[n-n_0]\} = z^{-n_0}X(z)$$
$$\mathcal{Z}\{x_1[n] * x_2[n]\} = X_1(z)X_2(z)$$
där $\mathcal{Z}\{x_1[n]\} = X_1(z)$ och
$\mathcal{Z}\{x_2[n]\} = X_2(z)$
$$\Rightarrow \mathcal{Z}\{x[n] * x[n-n_0]\} = z^{-n_0}\cdot X^2(z)$$

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2.

$$H(s) = \frac{3}{s^2+3s+3}$$

Poler $s_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4}-3}$

Komplexa! Kvadrat-kompletera.

$$H(s) = \frac{3}{\left(s+\frac{3}{2}\right)^2+3-\frac{9}{4}} =$$

$$= \frac{3}{(s+1{,}5)^2+0{,}75} = \frac{3}{\sqrt{0{,}75}} \cdot \frac{\sqrt{0{,}75}}{(s+1{,}5)^2+(\sqrt{0{,}75})^2}$$

Impulssvar $h(t) = \mathcal{L}^{-1}\{H(s)\} = \ldots = 2\sqrt{3}\, e^{-1{,}5t} \sin(0{,}866t)\, u(t)$

Stegsvar: $Y(s) = X(s) \cdot H(s)$ Insignal $x(t) = u(t)$
$\Rightarrow X(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s} \cdot \frac{3}{s^2+3s+3} = \frac{A}{s} + \frac{Bs+C}{s^2+3s+3}$$

$3 = A(s^2+3s+3) + s(Bs+C)$

s^0: $3 = A \cdot 3 \Rightarrow A = 1$

s^1: $0 = 3A + C \Rightarrow C = -3$

s^2: $0 = A + B \Rightarrow B = -1$

$$Y(s) = \frac{1}{s} - \frac{s+3}{(s+1{,}5)^2+0{,}75} = \frac{1}{s} - \frac{s+1{,}5+1{,}5}{(s+1{,}5)^2+0{,}75}$$

$$Y(s) = \frac{1}{s} - \frac{s+1{,}5}{(s+1{,}5)^2+(\sqrt{0{,}75})^2} - \frac{1{,}5}{\sqrt{0{,}75}} \cdot \frac{\sqrt{0{,}75}}{(s+1{,}5)^2+(\sqrt{0{,}75})^2}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[1 - e^{-1{,}5t}\left(\cos(0{,}866t) - \sqrt{3}\sin(0{,}866t)\right)\right] u(t)$$

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3.

$$y[n] = -y[n-1] + y[n-2] + x[n]$$

$$y[n] + y[n-1] - y[n-2] = x[n]$$

z-transf.

$$Y(z) + z^{-1}Y(z) - z^{-2}Y(z) = X(z)$$

$$Y(z)\left(1 + z^{-1} - z^{-2}\right) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + z^{-1} - z^{-2}} = \frac{z^2}{z^2 + z - 1}$$

Poler: $z_{1,2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 1} = -\frac{1}{2} \pm \sqrt{\frac{5}{4}} = \begin{cases} 0{,}618 \\ -1{,}618 \end{cases}$

PBU: $\frac{H(z)}{z} = \frac{z}{(z - 0{,}618)(z + 1{,}618)} = \frac{A}{z - 0{,}618} + \frac{B}{z + 1{,}618}$

$$A = \frac{0{,}618}{0{,}618 + 1{,}618} = 0{,}276 \qquad B = \frac{-1{,}618}{-1{,}618 - 0{,}618} = 0{,}724$$

$$H(z) = 0{,}276 \frac{z}{z - 0{,}618} + 0{,}724 \frac{z}{z + 1{,}618}$$

$$h[n] = \mathcal{Z}^{-1}\{H(z)\} = \left\{0{,}276\,(0{,}618)^n + 0{,}724\,(-1{,}618)^n\right\} u[n]$$

Instabilt ty kausalt system och pol utanför enhetscirkeln.

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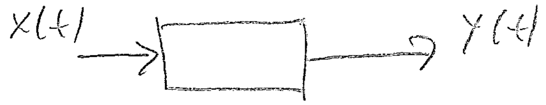
4. $H(s) = \dfrac{4000}{2000 + s}$ Frekv. svar $s = j\omega$

a) $H(j\omega) = \dfrac{4000}{2000 + j\omega} = \dfrac{2}{1 + j\frac{\omega}{\omega_0}}$ med $\omega_0 = 2000$ r/s



I stationärtillstånd – systemet påverkar amplitud och fas hos sinusformade signaler

$x(t) = 5{,}0 \cos(\omega t)$ med $\omega = 2500$ r/s

$y(t) = 5{,}0\,|H(j\omega)| \cos\left(\omega t + \arg\{H(j\omega)\}\right)$, $\omega = 2500$ r/s

$H(j\omega)\big|_{\omega=2500} = \dfrac{2}{\sqrt{1 + \left(\frac{2500}{2000}\right)^2}} \approx 1{,}25$

$\arg\{H(j\omega)\}_{\omega=2500} = -\arctan\left(\dfrac{2500}{2000}\right) \approx -51{,}8^\circ$

Svar: $y(t) = 6{,}25 \cos(2500t - 51{,}3^\circ)$

b) $h(t) = \mathcal{L}^{-1}\{H(s)\} = 4000\, e^{-2000t}\, u(t)$

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5.

$f_1 = 329{,}6$ Hz
$f_2 = 246{,}9$ Hz

Samplingsfrekvens $f_s = \frac{1}{T} = \frac{1}{200\mu s} = 5000$ Hz

Frekvensupplösning

$\Delta f = \frac{f_s}{N}$ svarar mot $(k+1) - k$ i index hos DFT

$f_1 \curvearrowright k_1$ i DFT
$f_2 \curvearrowright k_2$ i DFT

$k_1 - k_2 > 8$

$8 \cdot \Delta f < f_1 - f_2$

$8 \cdot \frac{f_s}{N} < f_1 - f_2$

$N > 8 \frac{f_s}{f_1 - f_2} = \frac{8 \cdot 5000}{329.6 - 246.9} \approx 484$

Svar: $N > 484$