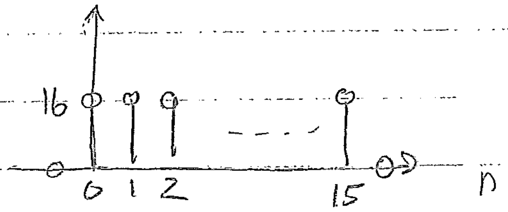
Signaler och System, Z2

SSY042 2017-06-08

1a)

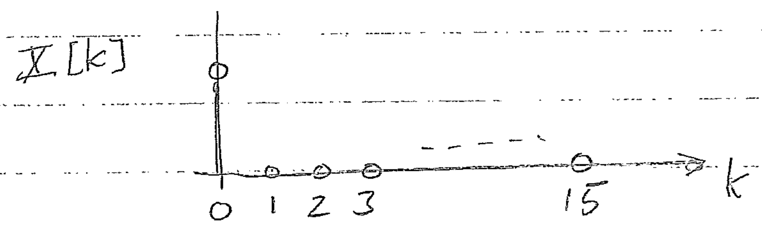
$x_a[n] = N, \quad n = 0,1,2,\ldots,N-1, \quad N = 16$



$$x_a[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi}{N}kn} \qquad (1)$$

$n = 0,1,2,\ldots,N-1$

$x_a[n]$ innehåller inga sinusformade svängningar $e^{j\frac{2\pi}{N}kn}$, utan endast en "DC" komponent $\Rightarrow$ $x_a[n]$ får bidrag från ekv. 1 endast för $k=0$ och $X[0] = N^2$.



$$X[k] = \begin{cases} N^2 = 16^2, & k=0 \\ 0, & k=1,2,\ldots,N-1 \end{cases}$$

c) Frekvensupplösning $\Delta\Omega = \frac{2\pi}{N}$ rad

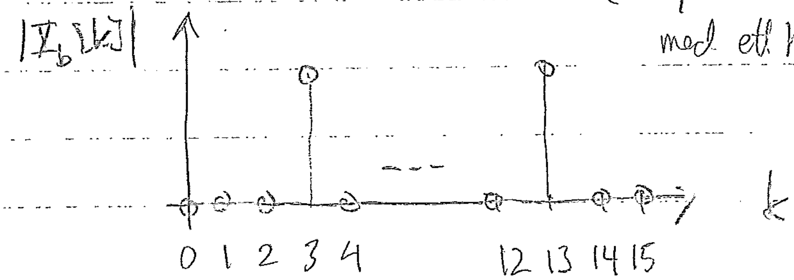
$\omega = \frac{\Omega}{T} \Rightarrow \Delta\omega = \frac{\Delta\Omega}{T} = \frac{2\pi}{N \cdot 1{,}25\cdot 10^{-4}} = \{N=16\} = 1000\pi$ rad/s

Svar: 1000π rad/s

b) $\frac{\text{Aktuell frekvens}}{\text{Frekvensupplösn.}} = \frac{\omega}{\Delta\omega} = \frac{3000\pi}{1000\pi} = 3$

$\Rightarrow X_b[k]$ bidrag vid $k=3$ samt $N-3 = 16-3 = 13$

(ty Reell sinusformad signal med ett helt antal perioder i intervallet.)



2. $$LC\frac{d^2u_L(t)}{dt^2}+RC\frac{du_L(t)}{dt}+u_L(t)=LC\frac{d^2u_0(t)}{dt^2}$$

$R=4.8\,\Omega$
$L=4.0\,H$
$C=0.25\,F$

Laplacetransformera! (Inga beg. energier).

$$LC\,s^2U_L(s)+RC\cdot s\,U_L(s)+U_L(s)=LC\,s^2\,U_0(s)$$

$$U_L(s)=U_0(s)\frac{LCs^2}{LCs^2+RCs+1}=U_0(s)\frac{s^2}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$u_0(t)=\begin{cases}4{,}0\text{ V}, & t\geq 0\\ 0\text{ V}, & t<0\end{cases}\Rightarrow U_0(s)=\frac{4}{s}$$

komplexa poler!

$$U_L(s)=\frac{4}{s}\cdot\frac{s^2}{s^2+1{,}2s+1}=4\frac{s}{(s+0{,}6)^2+1-0{,}6^2}=$$

$$=4\frac{s}{(s+0{,}6)^2+0{,}8^2}=4\left\{\frac{s+0{,}6}{(s+0{,}6)^2+0{,}8^2}-\frac{0{,}6}{(s+0{,}6)^2+0{,}8^2}\right\}=$$

$$=4\left\{\frac{s+0{,}6}{(s+0{,}6)^2+0{,}8^2}-\frac{0{,}6}{0{,}8}\cdot\frac{0{,}8}{(s+6)^2+0{,}8^2}\right\}$$

Inverstransformera

$$u_L(t)=e^{-0{,}6t}\left(4\cos(0{,}8t)-3\sin(0{,}8t)\right),\quad t\geq 0$$

SSY042

2017-06-08

3. $y[n] - 0{,}6\,y[n-1] + 0{,}08\,y[n-2] = 2x[n] - 0{,}4\,x[n-1]$

z-transformera

$$Y(z)\left[1 - 0{,}6z^{-1} + 0{,}08z^{-2}\right] = X(z)\left[2 - 0{,}4z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2(1-0{,}2z^{-1})}{1-0{,}6z^{-1}+0{,}08z^{-2}} = \frac{2z(z-0{,}2)}{z^2-0{,}6z+0{,}08}$$

Poler: $z_{1,2} = 0{,}3 \pm \sqrt{0{,}3^2 - 0{,}08} = 0{,}3 \pm 0{,}1 = \begin{cases} 0{,}4 \\ 0{,}2 \end{cases}$

$$H(z) = \frac{2z(z-0{,}2)}{(z-0{,}2)(z-0{,}4)} = \frac{2z}{z-0{,}4}$$

$$x[n] = (-0{,}8)^n u[n] \rightarrow X(z) = \frac{1}{1+0{,}8z^{-1}} = \frac{z}{z+0{,}8}$$

$$Y(z) = H(z)\cdot X(z) = \frac{2z^2}{(z-0{,}4)(z+0{,}8)} = z \cdot \underbrace{\frac{2z}{(z-0{,}4)(z+0{,}8)}}_{\text{P.B.U.}}$$

$$\frac{2z}{(z-0{,}4)(z+0{,}8)} = \frac{A}{z-0{,}4} + \frac{B}{z+0{,}8} \Rightarrow 2z = A(z+0{,}8) + B(z-0{,}4)$$

$z = -0{,}8 \Rightarrow 2(-0{,}8) = B(-1{,}2) \qquad B = \frac{4}{3}$

$z = 0{,}4 \Rightarrow 2\cdot(0{,}4) = A(1{,}2) \qquad A = \frac{2}{3}$

$$Y(z) = \frac{2}{3}\cdot\frac{z}{z-0{,}4} + \frac{4}{3}\cdot\frac{z}{z+0{,}8} \overset{\mathcal{Z}^{-1}}{\Rightarrow} y[n] = \left\{\frac{2}{3}(0{,}4)^n + \frac{4}{3}(-0{,}8)^n\right\}u[n]$$

$$= \frac{2}{3}\left\{(0{,}4)^n + 2(-0{,}8)^n\right\}u[n]$$

4.

SSY042
2017-06-08

$x(t) = \cos(\omega_c t) + \cos(1{,}6\,\omega_c t)$ Fouriertransformera:

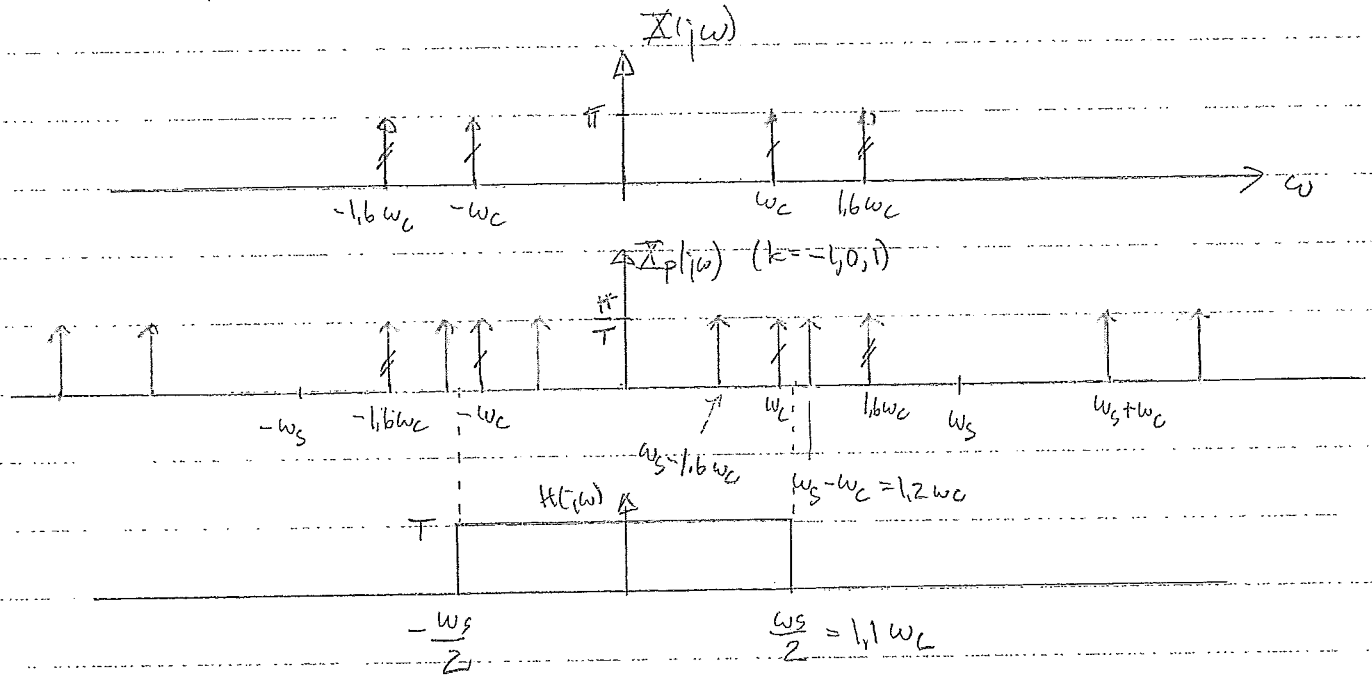
$$X(j\omega) = \pi\big(\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\big) + \pi\big(\delta(\omega-1{,}6\omega_c) + \delta(\omega+1{,}6\omega_c)\big)$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) \xleftrightarrow{FT} P(j\omega) = \frac{2\pi}{T}\sum \delta(\omega - k\omega_s)$$

$$\begin{matrix} x_p(t) \\ X_p(j\omega)\end{matrix} \longrightarrow \boxed{H(j\omega)} \longrightarrow \begin{matrix} y(t) \\ Y(j\omega)\end{matrix}$$

$$x_p(t) = x(t)\cdot p(t) \xleftrightarrow{FT} X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) =$$

$$= \frac{1}{2\pi}\cdot\frac{2\pi}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$



$$Y(j\omega) = H(j\omega)\cdot X_p(j\omega) = \pi\big(\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\big) +$$
$$+ \pi\Big(\delta\big(\omega - \underbrace{(\omega_s - 1{,}6\omega_c)}_{0{,}6\omega_c}\big) + \delta\big(\omega + \underbrace{(\omega_s - 1{,}6\omega_c)}_{0{,}6\omega_c}\big)\Big)$$

Inv. transf. ger

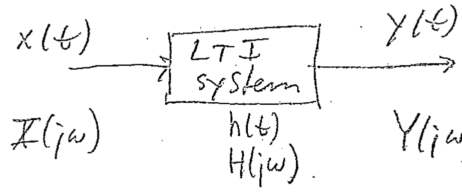
$$y(t) = \cos(\omega_c t) + \cos(0{,}6\,\omega_c t)$$

Aliasing!

5.

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{k}{1+k^2} e^{jk4t} \qquad (\omega_0 = 4)$$

$$x_1(t) = e^{jk\omega_0 t} \overset{FT}{\longleftrightarrow} 2\pi\delta(\omega - k\omega_0) = X_1(j\omega)$$



$$Y(j\omega) = H(j\omega)X(j\omega)$$

Insignal transf. Utsignal transf.

$$X_1(j\omega) = 2\pi\delta(\omega - k\omega_0) \qquad H(j\omega)\,2\pi\delta(\omega - k\omega_0) = 2\pi H(jk\omega_0)\delta(\omega - k\omega_0)$$

$$\Rightarrow y_1(t) = H(jk\omega_0)\, e^{jk\omega_0 t}$$

Superpos. ger

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_0) \frac{k}{1+k^2} e^{jk4t}$$

$$h(t) = \delta(t) - e^{-3t}u(t) \overset{FT}{\longleftrightarrow} H(j\omega) = 1 - \frac{1}{3+j\omega} = \frac{3+j\omega-1}{3+j\omega} =$$

$$= \frac{2+j\omega}{3+j\omega}$$

Utsignalens kompletta Fourierserie

$$y(t) = \sum_{k=-\infty}^{\infty} H(jk\omega_0) \frac{k}{1+k^2} e^{jk\omega_0 t} = \left\{ \omega_0 = 4,\ H(j\omega) = \frac{2+j\omega}{3+j\omega} \right\} =$$

$$= \sum_{k=-\infty}^{\infty} \frac{2+jk4}{3+jk4} \cdot \frac{k}{1+k^2} e^{jk4t}$$