Sensorer, Signaler & System
SSY041 A 12-12-18

1 a) i) $g(t-t_0)$

ii) $g(t_0)\,\delta(t-t_0)$

iii) $g(-t_0)$

b) $$u(t) = L\frac{di(t)}{dt} + Ri(t)$$

Insignal	Utsignal
$i_1(t)$	$L\frac{di_1(t)}{dt} + Ri_1(t) = u_1(t)$
$i_2(t)$	$L\frac{di_2(t)}{dt} + Ri_2(t) = u_2(t)$
$ai_1(t)$	$L\frac{d}{dt}(ai_1(t)) + Rai_1(t) = au_1(t)$
$ai_1(t) + bi_2(t)$	$L\frac{d}{dt}(ai_1(t) + Rai_1(t) + L\frac{d}{dt}(bi_2(t) + Rbi_2(t) =$

$$= au_1(t) + bu_2(t)$$

Superposition!

Systemet är linjärt

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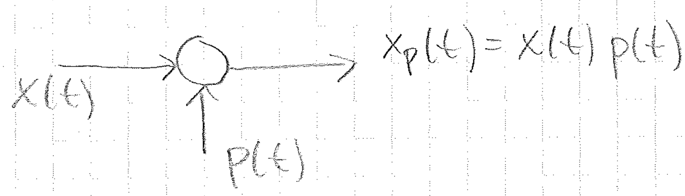
2/



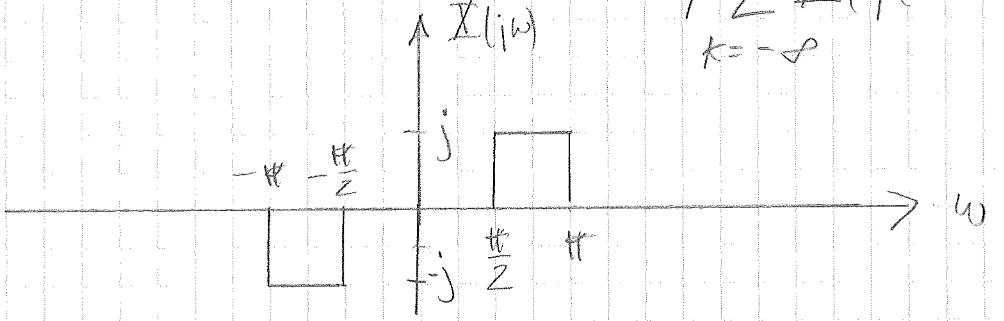
$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) \xleftrightarrow{FT} P(j\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(t-k\omega_s)$$

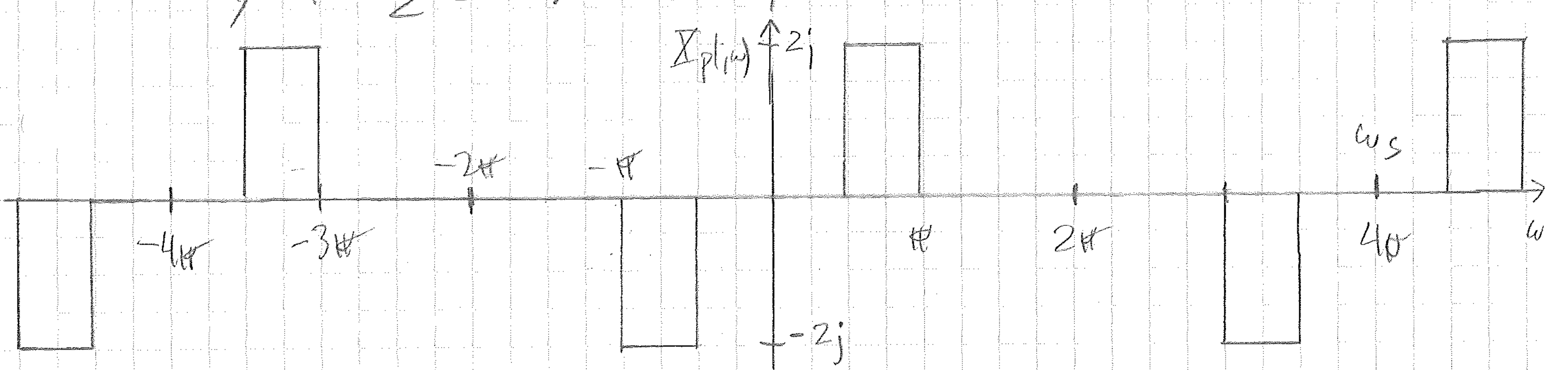
med $\omega_s = \frac{2\pi}{T}$

$$x_p(t) = x(t)\cdot p(t) \xleftrightarrow{FT} X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) =$$

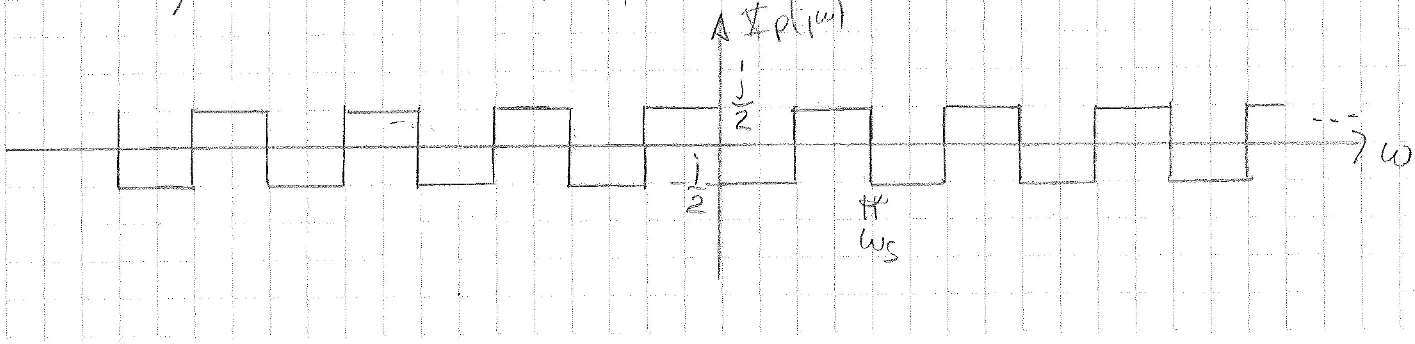
$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$



a) $T = \frac{1}{2}$ s $\Rightarrow$ $\omega_s = \frac{2\pi}{T} = 4\pi$



b) $T = 2$ s $\Rightarrow$ $\omega_s = \frac{2\pi}{T} = \pi$



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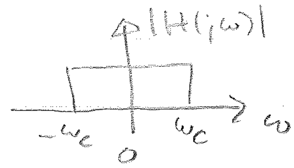
3/ $H(j\omega) = \begin{cases} e^{-j\omega t_0} & , |\omega| \leq \omega_c \\ 0 & , |\omega| > \omega_c \end{cases}$



$|H(j\omega)| = 1 \quad , \quad \arg\{H(j\omega)\} = -\omega t_0 \quad$ för $|\omega| \leq \omega_c$

$\omega_c = \frac{7\pi}{T} = \frac{7\pi}{2\pi} \cdot \omega_0 = $
$= 3{,}5\,\omega_0$

$$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow y(t)$$

a/ $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$ $\qquad c_0 = 0$
$\qquad c_k = \frac{2}{k^2} \, , \; k \neq 0$

$$x(t) \overset{FT}{\longleftrightarrow} X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi c_k \delta(\omega - k\omega_0)$$

$$Y(j\omega) = H(j\omega) \cdot X(j\omega) = \sum_{k=-3}^{3} 2\pi e^{-j\omega t_0} c_k \delta(\omega - k\omega_0) =$$

$$= 2\pi \sum_{k=-3}^{3} c_k e^{-jk\omega_0 t_0} \delta(\omega - k\omega_0)$$

$$y(t) = \sum_{k=-3}^{3} c_k^y e^{jk\omega_0 t} \quad \text{med } c_k^y = c_k \cdot e^{-jk\omega_0 t_0}$$

b/ Medeleffekt $\overline{P_y} = \frac{1}{T} \int_T |y(t)|^2 dt = \sum_{k=-3}^{3} |c_k^y|^2 =$

$$= \left\{ \omega_c = \frac{7\pi}{T} = \frac{7\pi}{2\pi} \cdot \omega_0 = 3{,}5\,\omega_0 \right\} = \sum_{k=-3}^{3} |c_k^y|^2 = \{c_0 = 0\}$$

$$= 2 \cdot \left(\frac{2}{1}\right)^2 + 2\left(\frac{2}{4}\right)^2 + 2 \cdot \left(\frac{2}{9}\right)^2 = 8 + \frac{1}{2} + \frac{8}{81} = 8{,}56$$

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4)

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

a) $x[n] = u[n]$

$h[n] = \beta^n u[n]$ $\qquad |\beta| < 1$

$$y[n] = \sum_{k=0}^{\infty} \beta^k u[n-k] =$$

$$= \sum_{k=0}^{n} \beta^k = \{\text{Ändlig geom. summa}\} =$$

$$= \frac{1-\beta^{n+1}}{1-\beta}, \quad n \geq 0$$

b) $x[n] = u[n-3]$

$y[n] = \underline{0}$ för $n-3 < 0 \Rightarrow \underline{\underline{n < 3}}$

$\underline{\underline{n \geq 3}}$, $y[n] = \sum_{k=0}^{n-3} \beta^k = \{\text{Ändlig geom. summa}\} =$

$$= \frac{1-\beta^{n-2}}{1-\beta}$$

Alt. $\quad x[n] \rightarrow y[n]$

$x[n-3] \rightarrow y[n-3]$

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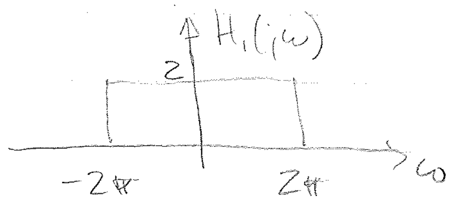
5/ $$h(t) = \underbrace{2\,\frac{\sin(2\pi t)}{\pi t}}_{h_1(t) \overset{FT}{\longleftrightarrow} H_1(j\omega)} \cdot \underbrace{\cos(7\pi t)}_{h_2(t) \overset{FT}{\longleftrightarrow} H_2(j\omega)}$$

$$h(t) = h_1(t) \cdot h_2(t) \overset{FT}{\longleftrightarrow} \frac{1}{2\pi} H_1(j\omega) * H_2(j\omega)$$

H_1: $$2\,\frac{\sin(2\pi t)}{\pi t} \overset{FT}{\longleftrightarrow} 2\left[u(\omega+2\pi) - u(\omega-2\pi)\right] = H_1(j\omega)$$

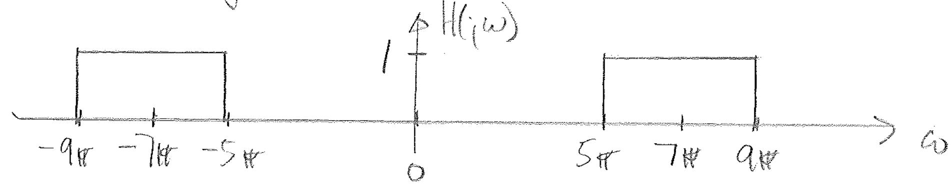
Se bela!



H_2: $$\cos(7\pi t) \overset{FT}{\longleftrightarrow} \pi\left[\delta(\omega+7\pi) + \delta(\omega-7\pi)\right]$$

$$\frac{1}{2\pi} H_1(j\omega) * H_2(j\omega) = \frac{1}{2\pi} H_1(j\omega) * \pi\left[\delta(\omega+7\pi) + \delta(\omega-7\pi)\right] =$$
$$= \frac{\pi}{2\pi}\left[H_1(j(\omega+7\pi)) + H_1(j(\omega-7\pi))\right] = H(j\omega)$$

Resultat enligt figur



b/ $x(t) = \cos(2\pi t) + \sin(6\pi t)$

$X(j\omega) = \pi\left[\delta(\omega+2\pi) + \delta(\omega-2\pi)\right] + \frac{\pi}{j}\left[\delta(\omega+6\pi) - \delta(\omega-6\pi)\right]$

$Y(j\omega) = H(j\omega) \cdot X(j\omega) \Rightarrow$ $\cos(2\pi t)$ filtreras bort
$\sin(6\pi t)$ passeras

$y(t) = \sin(6\pi t)$