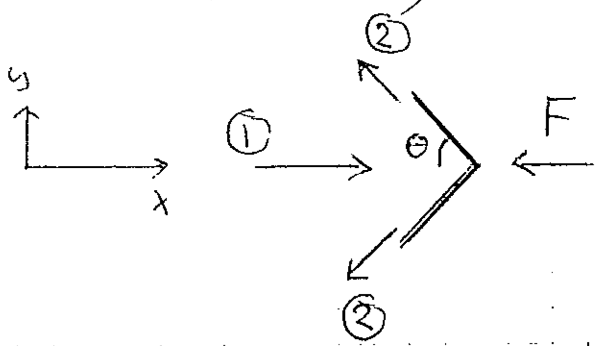
MTF052 Strömningsmekanik 12.01.11

impulssatsen

$$F = \dot{m}(\mathbb{V}_2 - \mathbb{V}_1)$$



x-led:

$$-F = \dot{m}(-V\cos\theta - V)$$

$$-F = -\dot{m}V(\cos\theta + 1)$$

$$F = \dot{m}V(\cos\theta + 1)$$

$$F = \rho A V^2(\cos\theta + 1)$$

$$= \rho \frac{D^2\pi}{4} V^2(\cos\theta + 1)$$

$$\cos\theta = \frac{F \cdot 4}{\rho D^2 \pi V^2} - 1$$

$$\theta = \arccos\left[\frac{4F}{\rho D^2 \pi V^2} - 1\right]$$

$$\theta = 24{,}5°$$

NS $$-\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} = 0$$

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2$$

RV: $u(0) = C_2 = -\frac{U_0}{2} = -5\,m/s$

$$u(h) = \frac{1}{\mu}\frac{dp}{dx}\frac{h^2}{2} + C_1 h - \frac{U_0}{2} = U_0$$

$$C_1 h = \frac{3}{2}U_0 - \frac{1}{\mu}\frac{dp}{dx}\frac{h^2}{2}$$

$$C_1 = \frac{3U_0}{2h} - \frac{1}{\mu}\frac{dp}{dx}\frac{h}{2} = -250\,s^{-1}$$

$$Q = \int_A u\,dA = \int_A u\,dy\,dz =$$

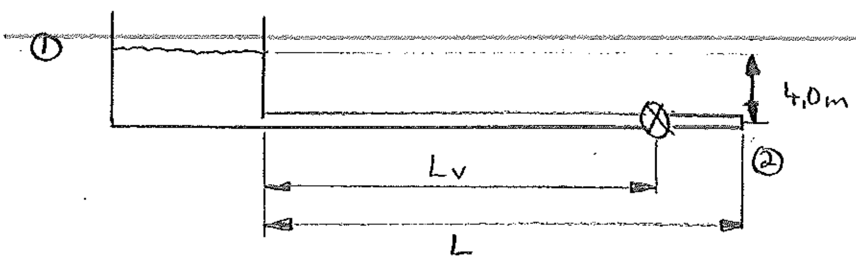
$$= dz\int_0^h \frac{1}{\mu}\frac{dp}{dx}\frac{y^2}{2} + C_1 y + C_2\,dy$$

$$\frac{Q}{dz} = \left[\frac{1}{\mu}\frac{dp}{dx}\frac{y^3}{6} + C_1\frac{y^2}{2} + C_2 y\right]_0^h$$

$$\frac{Q}{dz} = \frac{1}{\mu}\frac{dp}{dx}\frac{h^3}{6} + C_1\frac{h^2}{2} + C_2 h =$$

$$= -0{,}0375\,m^2/s$$

Svar: 37,5 L/ms i negativ x-riktning



Givet: $L_{rör} = 15\,m$, $L_v = 14\,m$, $P_1 = P_2 = P_{atm}$
$\nu = 0{,}5 \cdot 10^{-6}\,m^2/s$, $d = 0{,}08\,m$
Galvaniserat järn, Tabell 6.1 s.365 $\Rightarrow \varepsilon = 0{,}15\,mm$

$\frac{\varepsilon}{d} = \frac{0{,}15}{80} = 0{,}002$. Ventilen stängd efter $t = 4\,s$.

Sökt: Är $u_{max} \cdot t$ större eller mindre än 14 m?

Lösn: Bernoullis utv. ekv. (3.68b):

$$\cancel{P_1} + \cancel{\rho\frac{V_1^2}{2}} + \rho g z_1 = \cancel{P_2} + \rho\frac{V_2^2}{2} + \rho g z_2 + \Delta p_f + \cancel{\rho w_s}$$

stor tank — ingel tekn arb

$$\Rightarrow \Delta p_f = \rho g(z_1 - z_2) - \rho\frac{V_2^2}{2} \quad (1)$$

Friktion i röret + engångsförlust i ventil, (6.100b) $\Rightarrow$

$$\Delta p_f = \left(f\frac{L}{d} + K\right)\rho\frac{V_2^2}{2} \quad (2)$$

$$(1) \circ (2) \Rightarrow V_2 = \sqrt{\frac{2g(z_1 - z_2)}{f\frac{L}{d} + K + 1}} \quad (3)$$

Antag ventilen helt öppen i 4 sek, därefter stängd (ger lågt K, högt flöde, värsta fallet)

Tabell 6.5 s.385 $\Rightarrow K \approx 6$.
Vi söker $V_2 \Rightarrow$ iteration, Gissa $f = 0{,}025$

(3) med $L = L_{rör} = 15\,m \Rightarrow V_2 = 2{,}59\,m/s \Rightarrow$

$$\left.\begin{aligned} Re &= \frac{Vd}{\nu} = 4{,}15 \cdot 10^5 \\ \text{Moody-diag Fig 6.13 s.364}, \ \frac{\varepsilon}{d} &= 0{,}002 \end{aligned}\right\} \Rightarrow f = 0{,}024$$

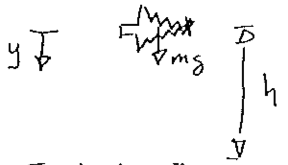
Ins i (3) $\Rightarrow V_2 = 2{,}61\,m/s$, $Re = 4{,}18 \cdot 10^5$
Moody $\Rightarrow f = 0{,}024$, stämmer.
$Re > 2300 \Rightarrow$ turbulent, (6.43b) $\Rightarrow$

$$u_{max} = \frac{V_2}{0{,}82} = \frac{2{,}61}{0{,}82} = 3{,}2\,m/s \Rightarrow$$

$$u_{max} \cdot t = 3{,}2 \cdot 4 = 12{,}8\,m < 14\,m$$

Svar: Ventilen hinner stängas.

GIVET:
$m = 15\,kg$
$h = 20\,m$
$A = 1\,m^2$



SÖKT:
Nedslagshastigheten.

LÖSNING:
Newton II: (Försumma flytkraften)

$$m\frac{du}{dt} = mg - \frac{1}{2}\rho u^2 C_D A \quad \left(\text{Tab. 7.3} \Rightarrow C_D \approx 1{,}2\right)$$

Skriv om:

$$\frac{du}{dt} = \frac{dy}{dt}\cdot\frac{du}{dy} = u\frac{du}{dy} \Rightarrow$$

$$mu\frac{du}{dy} = mg - \frac{1}{2}\rho u^2 C_D A$$

Separabel diffekv $\Rightarrow$

$$m\int_0^u \frac{u}{mg - \frac{1}{2}\rho u^2 C_D A}\,du = \int_0^h dy = h \Rightarrow$$

$$\Rightarrow \int_0^u \frac{u}{au^2 + b}\,du = \frac{h}{m}$$

där $a = -\frac{1}{2}\rho C_D A$ och $b = mg$.

Integralen finns i Beta, s. 145, nr 59

$$\Rightarrow \text{V.L} = \int_0^u \frac{u\,du}{au^2 + b} = \frac{1}{2a}\Big[\ln|au^2 + b|\Big]_0^u =$$

$$= \frac{1}{2a}\left\{\ln|au^2 + b| - \ln|b|\right\} = \frac{1}{2a}\ln\left\{\frac{|au^2 + b|}{|b|}\right\}$$

$$= -\frac{1}{\rho C_D A}\ln\left\{\frac{|mg - \frac{1}{2}\rho u^2 C_D A|}{mg}\right\} = \text{H.L} = \frac{h}{m}$$

$$\Rightarrow \frac{|mg - \frac{1}{2}\rho u^2 C_D A|}{mg} = e^{-\frac{\rho C_D A h}{m}}$$

Anta att $mg > \frac{1}{2}\rho u^2 C_D A \Rightarrow$

$$\frac{1}{2}\rho u^2 C_D A = mg\left(1 - e^{-\rho C_D A h/m}\right)$$

$$\Rightarrow u = \sqrt{\frac{2mg}{\rho C_D A}\left(1 - e^{-\frac{\rho C_D A h}{m}}\right)}$$

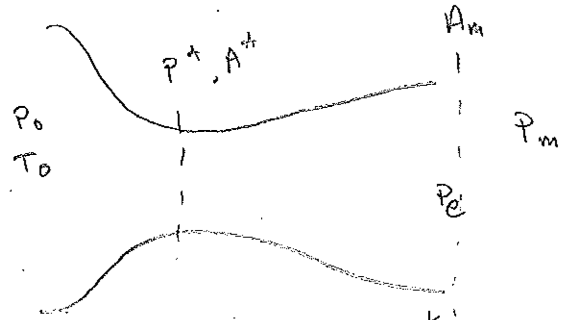
Sätt in siffror $\Rightarrow u \approx 13{,}2\,m/s$

Kontrollera antagandet: $mg = 15 \cdot 9{,}81 = 147{,}15\,N$
$\frac{1}{2}\rho u^2 C_D A = \frac{1}{2} \cdot 1{,}2 \cdot 13{,}2^2 \cdot 1{,}2 \cdot 1 = 125$,

$\therefore mg > \frac{1}{2}\rho C_D A$ och Lösn. OK.

Svar: $u = 13{,}2\,m/s$.

LÖSNING:



$$(9{,}32): \quad \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \approx 0{,}5283$$

$$\Rightarrow \quad p^* = 0{,}5283 \cdot 0{,}7 \cdot 10^6 = 0{,}3698 \cdot 10^6 \text{ MPa}$$

$p^* > p_m = 0{,}1$ MPa $\Rightarrow$ strömn. kritisk

$$(9{,}41): \quad \dot{m}_{max} = \frac{0{,}6847\, p_0 A^*}{\sqrt{RT_0}} = \frac{0{,}6847 \cdot 0{,}7 \cdot 10^6 \cdot \frac{\pi}{4} \cdot 0{,}005^2}{\sqrt{287 \cdot 303}} \approx 0{,}0320 \text{ kg/s}$$

Anta att ingen stöt inträffar i den divergerande delen. Då gäller enl. (9.45) och (9.28a):

$$\frac{A_m}{A^*} = \frac{1^2}{0{,}5^2} = 4 = \frac{1}{Ma_m} \frac{(1+0{,}2 Ma_m^2)^3}{1{,}728}$$

Tabell B1 $\Rightarrow$ $Ma_m = 2{,}94$

Med detta Ma-tal i mynningen skulle mynningstrycket, p_e bli:

$$\frac{p_0}{p_e} = \left[1 + \frac{1}{2}(k-1) Ma_m^2\right]^{\frac{k}{k-1}} \approx 33{,}56$$

$$\Rightarrow p_e \approx 0{,}0209 \text{ MPa}$$

Men $p_e < p_m = 0{,}1$ MPa och alltså fås en stöt i dysan (Se Fig. 9.12)