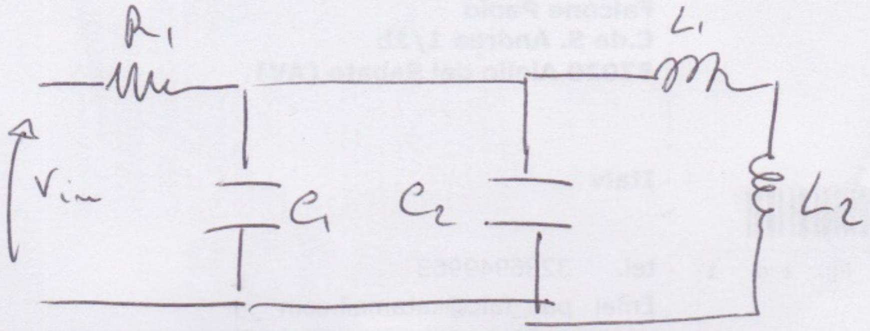
PROBLEM 1

1)



2) The resulting DAE is

$$\begin{bmatrix} RC_1 & RC_2 & 0 & 0 \\ 0 & 0 & L_1 & L_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 1 & 0 & R_1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

with index 2

PROBLEM 2

a) Data sets used in plot (b) are less noisy than in plot (a). This leads to a smaller variance $\bar{E}[(\hat{\theta}_{10}-\theta^0)^2]$ where $\theta^0 = 1$ is the true value of the parameter

The variance in plot (c) is even smaller due to larger data sets (100 samples).

The above arguments are supported by the relationship

$$\bar{E}[(\hat{\theta}-\theta^0)^2] \propto \frac{1}{N}\cdot\lambda,$$ where λ is the noise variance

b) It can be found that $a = -0.6$ and $b = 0.3$ (2)
because the model has not bias error.
That is the chosen model structure is identical to the model that has generated data.
It can be shown that, in this case, the estimated parameter converges to the real one as $N \to \infty$

PROBLEM 3

Set $H(\omega) = \dfrac{1}{2+2\cos\omega t}$. Since $H(\omega) \simeq \Phi_y(\omega)$,

it holds that

$$H(\omega) \simeq |G(j\omega)|^2 \Phi_u(\omega) \quad \text{with } \Phi_u(\omega) = 0.2T$$

Hence:

$$|G(j\omega)|^2 \simeq \frac{1}{(2+2\cos\omega T')\,0.2T}$$

with

$$G(j\omega) = \frac{1 + \sum_{k=0}^{M_{num}} a_k e^{-kj\omega T'}}{\left(1 + \sum_{k=0}^{M_{den}} b_k e^{-kj\omega T'}\right)\sqrt{0.2T}}$$

setting $M_{num} = 0$, $a_0 = 0$, $M_{den} = 1$, $b_1 = 1$ solves the problem

PROBLEM 4

1) By simulating the system with the provided LMM, ~~obs~~ it can be seen that the solution oscillates, no matter how small the step size h is chosen, while it should have been constant and equal to 1

2) The zero-instability can be shown by studying the roots of the polynomial

$$\rho(r) = r^2 + 4r - 5$$

which are $r_1 = 1$ and $r_2 = 5$ clearly not inside the unitary circle

PROBLEM 5

1) $$\frac{x_k - x_{k-1}}{h} = A x_k + B u_k \Rightarrow x_k = (I - Ah)^{-1} x_{k-1} + (I - hA)^{-1} h B u_k$$

2) $$x_{k+1} = x_k + hA\left[x_k + \frac{h}{2}\left(A x_k + B u_k\right)\right] + hB u_k$$

PROBLEM 6

1) F

2) F

3) T

4) Frequency analysis, Fourier analysis, Spectral analysis

5) In explicit methods

$$x_k = G(t_k, x_{k-1}, \ldots)$$

In implicit methods

$$x_k = G(t_k, x_k, x_{k-1}, \ldots)$$