SOLUTIONS OF THE MODELING AND SIMULATION EXAM (ESS101)

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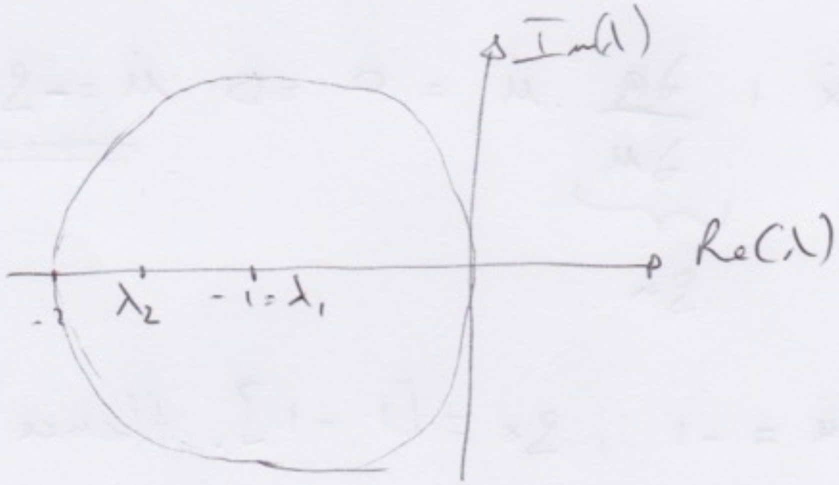
EX 1

(a) Rewrite the system as:

$$\dot{x} = Ax + Bu$$

The eigenvalues of A are $\lambda_1 = -1$, $\lambda_2 = -1.5$

A FE method with integration step size $T = 1$ can be used, because its stability region is



that clearly encloses λ_1 and λ_2.

Since $u(t)$ is not given at $t = 0, 1$, we assume $u(t) = 0 \ \forall t$ for simplicity. Hence

$$x(2) = A_d^2 \cdot x(0) = \begin{bmatrix} -0.25 \\ 0 \end{bmatrix} \quad \text{with } A_d = I + T \cdot A$$

(b) No, because the FE method solves systems of ODEs, while the addition of the algebraic equation $x_1(t) - x_2(t) = u(t)$ makes the system a DAE.

(e) The DAE

$$\dot{x} = Ax + Bu$$

$$x_1 - x_2 = u$$

is of the type $\begin{cases} \dot{x} = f(x,u) \\ g(x,u) = 0 \end{cases}$

that has index 1. To obtain a set of ODE 1 differentiation is required:

$$\underbrace{\frac{\partial g}{\partial x}}_{g_x} \cdot \dot{x} + \underbrace{\frac{\partial g}{\partial u}}_{g_u} \cdot \dot{u} = 0 \Rightarrow \underline{\underline{\dot{u} = -g_u^{-1} g_x \dot{x}}}$$

with $g_u = -1$, $g_x = [1 \ -1]$. Hence

$$\dot{u} = -1.5x_1 + 2x_2 - u$$

The resulting ODE is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{u} \end{bmatrix} = \begin{bmatrix} -1.5 & 1 & 0 \\ 0 & -1 & 1 \\ -1.5 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u \end{bmatrix}, \text{ with}$$

eigenvalues $\lambda_1 = -2.78$, $\lambda_2 = -0.72$, $\lambda_3 = 0$.

Let's integrate the system with FE and $T = 0.5$

We obtain

$$x(1) = \begin{bmatrix} 0.75 \\ 0.5 \\ 0.25 \end{bmatrix}, \quad x(2) = \begin{bmatrix} 0.4375 \\ 0.3750 \\ 0.0625 \end{bmatrix}$$

It's easy to check that $x_1(t) - x_2(t) = u(t)$ $\forall t$

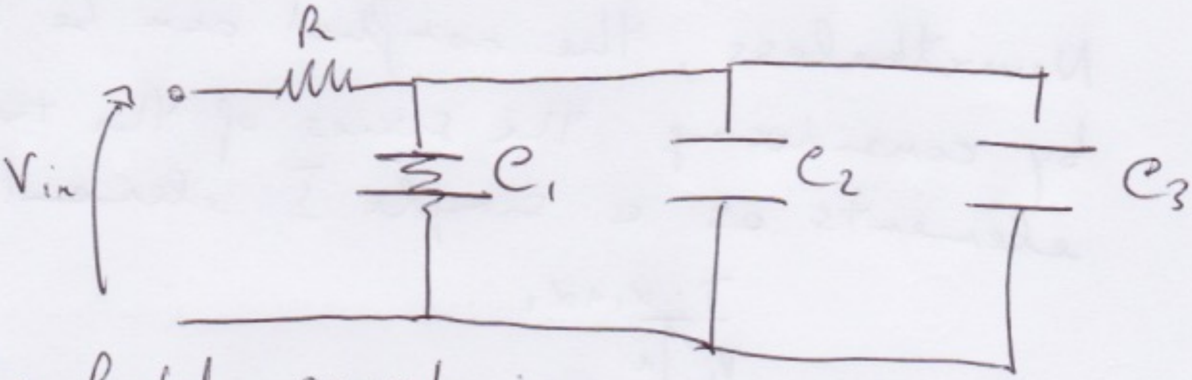
Ex 2

(a) $\hat{R}_{\varepsilon u}(\tau) \neq 0$ with $\tau < 0$ denotes the presence of feedback.

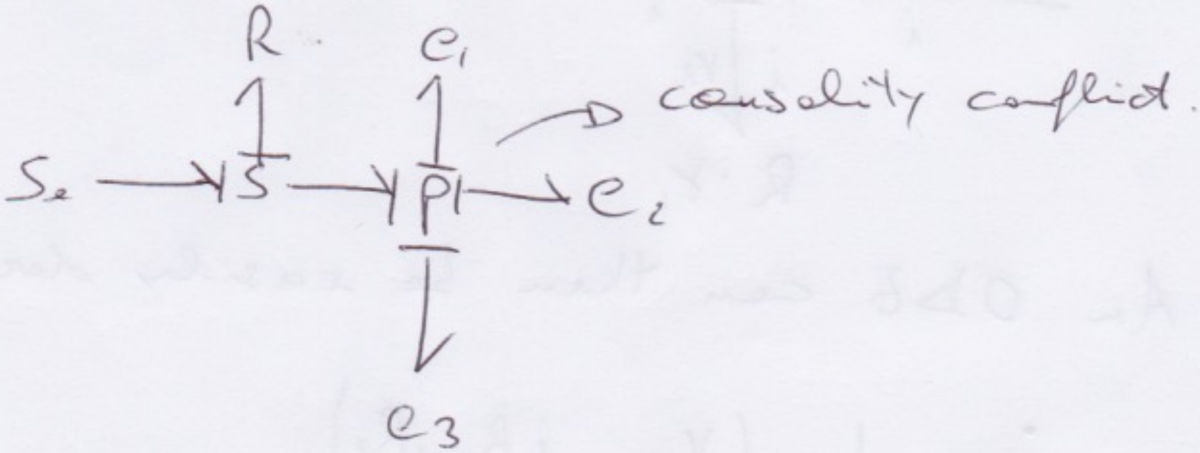
(b) $\hat{R}_{\varepsilon u}(\tau) \neq 0$ with $\tau > 0$ suggests a choice of a delay of τ / T_s steps in the model

Ex 3

(a)

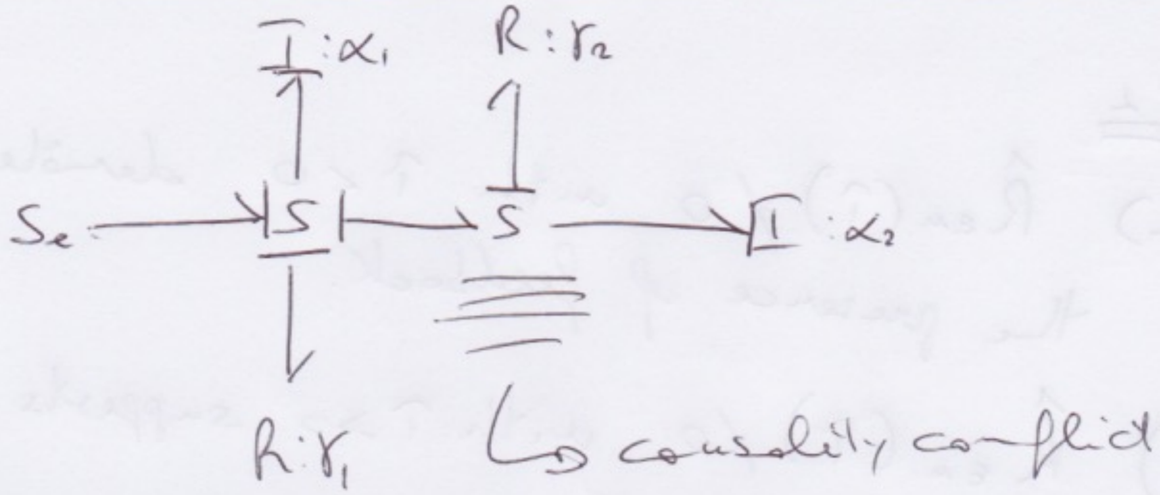


Its Bond graph is



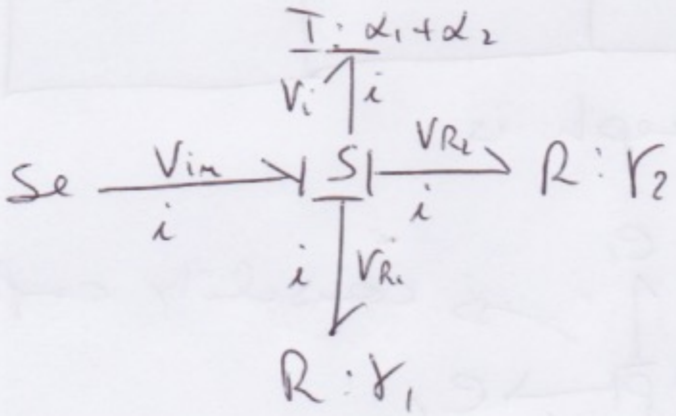
The causality conflict is due to the parallel of the three capacitors imposing the same voltages on them.

(b)



(c) Due to the causality conflict, the system at point (b) can't be written as an ODE as it is.

Nevertheless, the conflict can be removed by considering the series of the two I elements as a simple I element.



An ODE can then be easily derived

$$\dot{i} = \frac{1}{\alpha_1+\alpha_2}\left(V_{in} - iR_1 - iR_2\right)$$

Ex 4

1. False. The cost in the PEM in general is not convex

2. The periodogram is an estimate of the spectrum over a finite number of samples

3. The local error is the error introduced in one step of the iteration.
   The global error is the error introduced up to some simulation time.
   Here by error is meant the difference between the simulated and the true states

4. The LS formula can be applied if the input signal is persistently exciting

5.