SOLUTIONS OF THE MODELING AND SIMULATION EXAM (ESS101)

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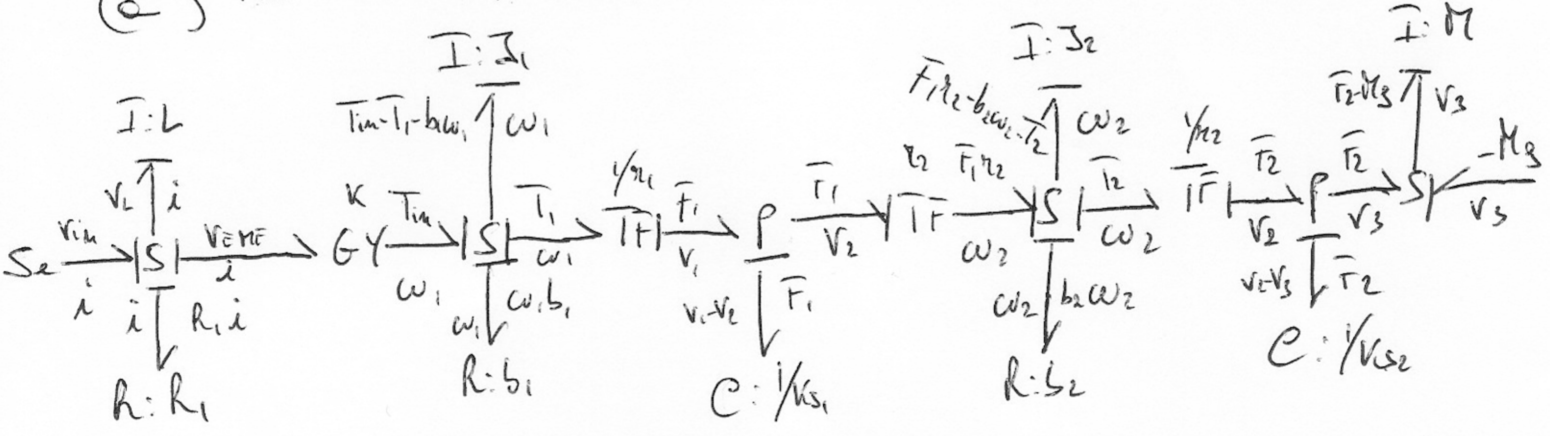
PROBLEM 1

(a) The bond graph of the system is:



(b) Since the bond graph of (a) does not have any causality conflict the state space model will be an ODE.

Define

$$x_1 = i\,,\quad x_2 = \int_0^t \omega_2(\tau)\,d\tau\,,\quad x_3 = \omega_1\,,\quad x_4 = \int_0^t \omega_2(\tau)\,d\tau\,,\quad x_5 = \omega_2$$

$$x_6 = \int_0 x\,,\quad x_7 = \dot{x}\,,\quad u = V_{in}$$

$$\dot{x}_1 = -\frac{R_1}{L}x_1 - \frac{K}{L}x_3 + \frac{1}{L}u$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = \frac{K}{J_1}x_1 - \frac{b_1}{J_1}x_3 - K_{s_1}(r_1x_2 - r_2x_4)$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = \frac{r_2}{J_2}K_{s_1}(r_1x_2 - r_2x_4) - \frac{b_2}{J_2}x_5 - K_{s_2}(r_2x_4 - x_6)$$

$$\dot{x}_6 = x_7$$

$$\dot{x}_7 = \frac{K_{s_2}}{r_2M}(r_2x_4 - x_6) - g$$

(e) If the two elastic springs are removed, the state variables are constrained as follows:

$$\omega_1 = \frac{r_2}{r_1}\omega_2 \quad \text{and} \quad \omega_2 r_2 = \dot{x}$$

These additional algebraic equations, with the previous constitutive laws, form a system of DAE.

PROBLEM 2

For a system, which is linear in the vector of parameters to estimate:

$$y(t) = \theta^T \varphi(t) + e(t)$$

the least squares formula is:

$$\hat{\theta} = \left[\frac{1}{N}\sum_{t=1}^{N} \varphi(t)\,\varphi^T(t)\right]^{-1} \cdot \frac{1}{N}\sum_{t=1}^{N} \varphi(t)\cdot y(t)$$

Hence, in order to apply the LS formula to our system, we can rewrite it as follows:

$$\tilde{y}(t) = \theta^T \varphi(t) + e(t) \quad \text{with} \quad \tilde{y}(t) = y(t) - 0.1\,y(t-1)$$

$$\theta = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$\varphi(t) = \begin{bmatrix} \cos y(t-2) \\ u(t) \end{bmatrix}$$

PROBLEM 3

The system considered in this problem has transfer function:

$$G(s) = \frac{\omega(s)}{T(s)} = \frac{1/b}{1 + \frac{I}{b}s}$$

where $\omega(s)$ and $T(s)$ are the Laplace transforms of the rotational speed and the input torque, respectively.

The spectrum $\Phi_\omega(\omega)$ of ω is:

$$\Phi_\omega(\omega) = |G(j\omega)|^2 \, \Phi_T(\omega) = |G(j\omega)|^2 = \frac{1/b^2}{1 + \frac{I^2}{b^2}\omega^2}$$

PROBLEM 4

1. TRUE
2. FALSE
3. FALSE
4. FALSE
5. FALSE