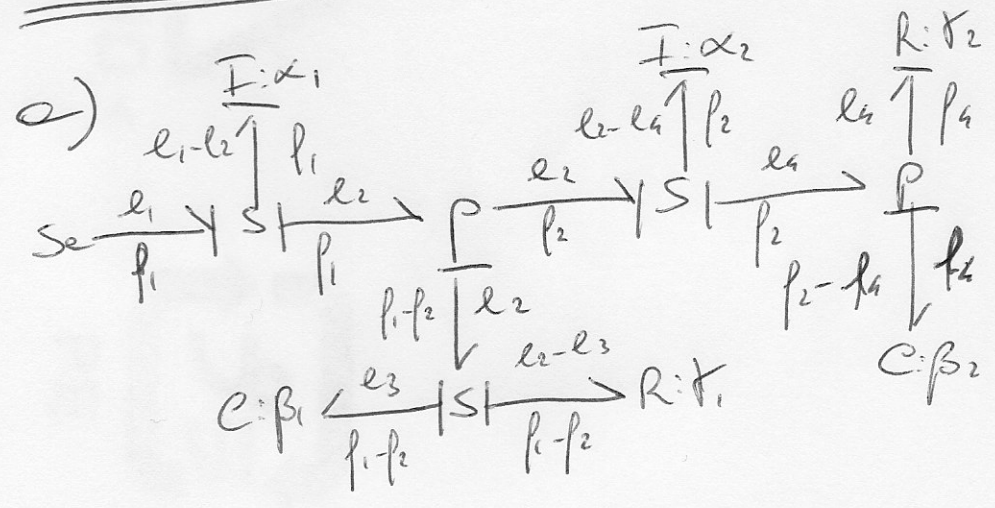


SOLUTIONS OF THE
ESS 101 - MODELING AND SIMULATION
EXAM

Examination date : 10/01/11

Teacher : Pedro Felcane

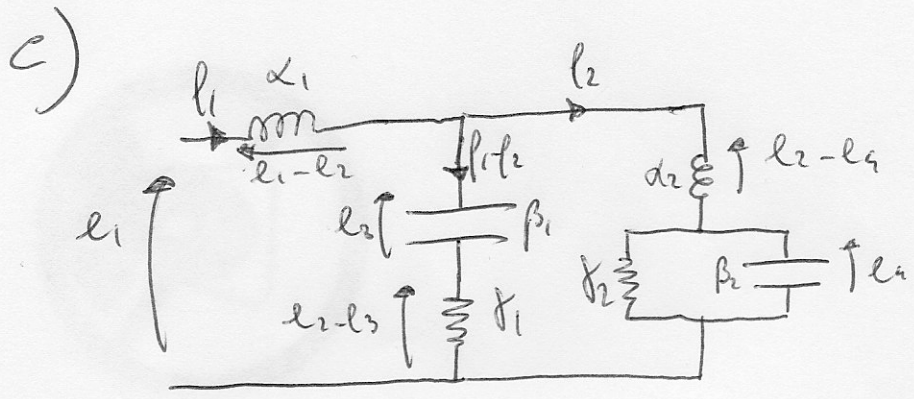
Problem 1



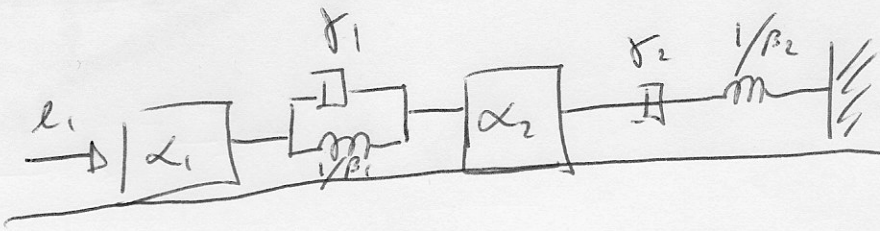
b) Define the state and the input vectors as

$$x = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} \quad u = l_1$$

$$\begin{cases} \dot{l}_1 = \alpha_1 (l_1 - l_2) \\ \dot{l}_3 = \beta_1 (l_1 - l_2) \\ \dot{l}_2 = \alpha_2 (l_2 - l_4) \\ \dot{l}_4 = \beta_2 (l_2 - l_4) \end{cases} \quad \begin{cases} \dot{x}_1 = -\gamma_1 \alpha_1 x_1 - \alpha_1 x_2 - \alpha_1 \gamma_1 x_3 + \alpha_1 u \\ \dot{x}_2 = \beta_1 x_1 - \beta_1 x_3 \\ \dot{x}_3 = \gamma_1 \alpha_2 x_1 + \alpha_2 x_2 - \gamma_1 \alpha_2 x_3 - \alpha_2 x_4 \\ \dot{x}_4 = \beta_2 x_3 - \frac{\beta_2}{\gamma_2} x_4 \end{cases}$$



d)



Problem 2

We next estimate the parameters of a model $A \times (1, 1)$.

By applying the least squares formula; the optimal vector of parameters \mathcal{J}_N^* is given by:

$$\mathcal{J}_N^* = \frac{1}{N} \sum_{t=1}^N \begin{pmatrix} y^2(t-1) & y(t-1)u(t) \\ y(t-1)u(t) & u^2(t) \end{pmatrix}^{-1} \begin{pmatrix} y(t-1)u(t) \\ u(t)y(t) \end{pmatrix}$$

As $N \rightarrow \infty$

$$\mathcal{J}_N^* \rightarrow \begin{pmatrix} R_y(0) & R_{yu}(1) \\ R_{yu}(1) & R_u(0) \end{pmatrix}^{-1} \begin{pmatrix} R_y(1) \\ R_{uy}(0) \end{pmatrix}$$

$$R_y(0) = E[y^2(t)] = E[+0.25y^2(t-1) + u^2(t) + e^2(t)]_2$$

$$= +0.25R_y(0) + 3 \Rightarrow R_y(0) = \cancel{0.090} \quad \leftarrow$$

$$R_{yu}(1) = 0, \quad R_u(0) = 2, \quad R_{uy}(0) = 2$$

$$R_y(1) = -0.5R_y(0) = \cancel{0.045} = 2$$

Hence

$$\Theta_n^* \rightarrow \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 1 \end{pmatrix}$$

The chosen parametrization is the correct one, i.e., Θ_n^* tends to the actual values as $N \rightarrow \infty$

Problem 3

a) In this case, the Backward Euler method can be used. The forward Euler method can't be used since, with the chosen integration step size, the eigenvalues of the system (-3 and -1) would not lie within its stability region.

b) Define the system

$$\dot{x} = Ax + Bu \quad \text{with}$$

$$A = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x_{k+1} = (I + hA)x_k + hB u_k \quad (\text{Forward Euler})$$

$$x_{k+1} = (I - hA)^{-1} x_k + (I - hA)^{-1} hB u_{k+1} \quad (\text{Backward Euler})$$

Problem 4

- 1) True .
- 2) False .
- 3) False
- 4) True
- 5) True