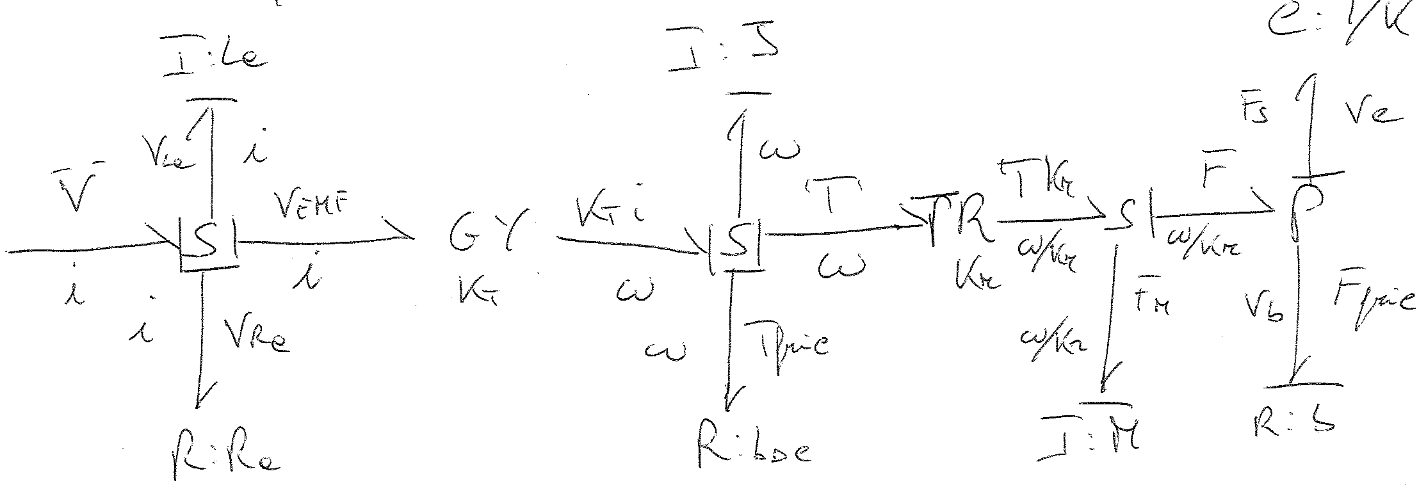
EXERCISE 1

a) The bond graph of the electromechanical system is:



Causality conflict at the third series junction

The following symbols have been introduced

L_a, R_e inductance, resistance of the DC-motor armature

V_{EMF} DC-motor back-emf

J DC-motor shaft inertia

b_{DC} DC-motor friction

b) The model in the state space is:

$$\begin{cases} \dot{i} = -\frac{R_e}{L_a} i - \frac{K_v}{L_a}\omega + \frac{\bar{V}}{L_a} \\ \dot{\omega} = \frac{K_T}{J} i - \frac{b_{DC}\omega}{J} - \frac{T}{J} \\ \dot{v} = \frac{T K_r}{m} - \frac{K}{m} x \\ \dot{x} = v - \frac{K}{b} x \\ \frac{\omega}{K_r} = v \end{cases}$$

This is a system of differential and algebraic equations (DAE)

EXERCISE 2

a) The system, under the considered feedback law, is:

$$y(t) + a\,y(t-1) = -b K_p \left[r(t-1) - y(t-1) \right] + v(t)$$

$$y(t) = -(a - bK_p)\, y(t-1) - bK_p\, r(t-1) + v(t)$$

The predictor can be written as:

$$\hat{y}(t,\vartheta) = \vartheta^T \varphi(t) \quad \text{with} \quad \vartheta = \begin{bmatrix} a - bK_p \\ bK_p \end{bmatrix}, \quad \varphi(t) = \begin{bmatrix} -y(t-1) \\ -r(t-1) \end{bmatrix}$$

Clearly, for $r(t-1) = 0 \;\forall t$, a and b cannot be estimated for a given K_p.

b) ~~The~~ A mathematical model of the system is:

$$G(s) = \frac{3}{1+0.2s} e^{-s}.$$

In particular, from the step response the system is clearly described by a first order model with a delay:

$$G(s) = \frac{K}{1+s\tau} e^{-\tau_d s}$$

- $\tau_d = 1$, since the step response starts at 2 s while the input step rises at 1 s

- $K = 3$, since the steady-state value of the output is 3
- $\tau = 0.25$ approximatively. This can be found by drawing the tangent to the step response in $t = 2s$ ~~[illegible]~~. τ is found as the time instant ~~[illegible]~~ when the tangent intersect the line $Y = 3$

EXERCISE 3

The process $w(t)$ can be described as:

$$W(s) = H(s)E(s)$$

Denote by $\Phi_w(\omega)$ and $\Phi_e(\omega)$ the spectra of $w(t)$ and $e(t)$, respectively. We have:

$$\Phi_w(\omega) = |H(i\omega)|^2 \Phi_e(\omega)$$

with $\Phi_e(\omega) = \lambda_e = 1$ since $e(t)$ is a white noise with variance λ_e

a) Since:

$$\Phi_w(\omega) = \frac{2}{2+\omega^2}$$

$$H(i\omega) = \frac{\sqrt{2}}{\sqrt{2}-i\omega}$$

b) Since

$$\Phi_w(i\omega) = \frac{1}{(\omega^2+2)(\omega^2+1)}$$

$$H(i\omega) = \frac{1}{(i\omega+\sqrt{2})(i\omega+1)}$$

EXERCISE 4

a) The system is an ARMAX(2,2)

b) The predictor for a system in the general form:

$$y(t) = G(\theta,q)\,u(t) + H(\theta,q)\,e(t)$$

is:

$$\hat{y}(t,\theta) = \left[1 - H^{-1}(\theta,q)\right] y(t) + H^{-1}(\theta,q)\,G(\theta,q)\,u(t)$$

In our case:

$$\hat{y}(t,\theta) = \frac{-0.0410q^{-1} + 0.0072q^{-2}}{1 - 1.438q^{-1} + 0.588q^{-2}}\, y(t) + \frac{0.0807q^{-1} + 0.1070q^{-2}}{1 - 1.438q^{-1} + 0.588q^{-2}}\, u(t)$$

c) By multiplying both sides by $1 - 1.438q^{-1} + 0.588q^{-2}$ we obtain:

$$\hat{y}(t,\theta) = -0.0410y(t-1) + 0.0072y(t-2) + 0.0807u(t-1) + 0.1070u(t-2) + 1.438\hat{y}(t-1,\theta) - 0.588\hat{y}(t-2,\theta)$$

EXERCISE 5

The equations used for simulating the given system with the Forward Euler (FE) and Backward Euler (BE) methods are:

$$x_{k+1} = x_k + hAx_k \qquad (FE)$$

$$x_{k+1} = (I + hA)^{-1} x_k \qquad (BE)$$

with:

$$A = \begin{bmatrix} -2 & 2 \\ 1 & 0 \end{bmatrix}$$

h the integration step.

For the FE method h must be selected in order to have numeric stability.

We observe that A has two eigenvalues

$$\lambda_1 = -2.7321 \quad \text{and} \quad \lambda_1 = 0.7321$$

In order to map the stable pole λ_1 into a stable discrete time pole, h must be selected in order to satisfy the inequality:

$$|1 + h\lambda| < 1$$

Hence $h \leq 0.73$