Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Examination date 090112

Time: 14.00 – 18.00

Teacher: Paolo Falcone, 772 1803

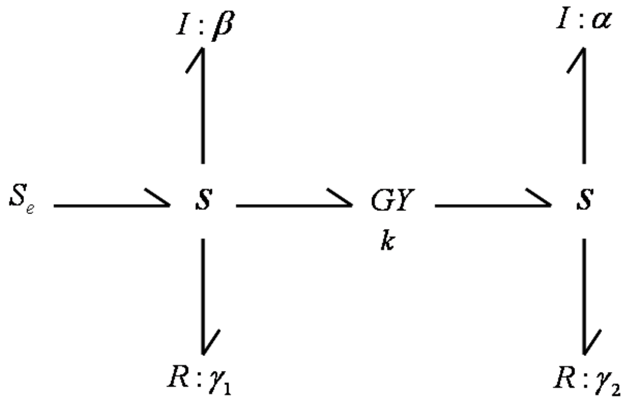
Allowed material during the exam: Mathematics Handbook.

The exam consists of 4 exercises of a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Jan 22. You can check the grading of your exam on Jan 23 at 12.30-13.15 at the Department of Signals and Systems.

Exercise 1 (5 p)

Consider the bond graph in the figure below.



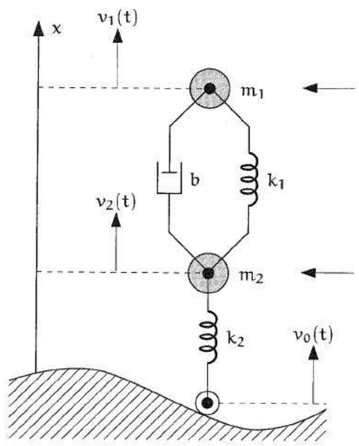
(a) Mark the causality of the graph. (1p)

(b) Derive an electro mechanical system from the bond graph in the figure. (2p)

(c) For the obtained system, formulate a state space model. (2p)

Exercise 2 (10 p)

Consider the car suspension system in the figure below,



where m_1 and m_2 are the masses of the vehicle and the wheel, respectively, k_1 and $b = b(u)$ are the spring and friction constants, respectively, of the suspension, u is an exogenous signal and k_2 is the spring constant of the wheel.

(a) Derive a state space model of the whole suspension system. (5p)

(b) Derive a state space model of the suspension system under the simplification of rigid wheel. (2p)

(c) For the model at point **(b)** of this exercise, assume $\dot{v}_0(t)$ is a white noise with variance 1. Compute the spectrum of the signal $\dot{v}_1(t)$. (3p)

Exercise 3 (5 p)

(a) Consider the system

$$y(t) = 0.5u(t-1) + \frac{1}{1+dz^{-1}}\xi(t)$$

where $\{\xi(t)\}$ is white noise with variance λ^2. Show how the least squares method can be applied to estimate the parameter d. (3p)

(b) Consider the system

$$y(t) = ay(t-1)^2 + b_1u(t-1) + b_2u(t-2)^3 + \xi(t)$$

where $\{\xi(t)\}$ is white noise with variance λ^2. Show how the least squares method can be applied to estimate the parameters a, b_1 and b_2. (2p)

Exercise 4 (5 p)

(a) Derive the stability regions of the forward and backward Euler integration methods. (3p)

(b) Choose the integration step size for integrating the following system of differential equations with the two methods. Motivate the answer. (2p)

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ 0 & -1 \end{bmatrix} x(t)$$