Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Examination date 080114

Time: 14.00 – 18.00

Teacher: Jonas Sjöberg, 772 1855

Allowed material during the exam: Mathematics Handbook.

The exam consists of 5 exercises of a total of 25 points. Nominal grading according to 12/17/21 points, you need 12 points to pass the course with grade 3, 17 points to pass with grade 4 and 21 to pass the course with grade 5. Solutions and answers should be written in English and be unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Jan 28. You may check your grading of your exam on Jan 29 at 12.30-13.15 at the Department of Signals and Systems.

GOOD LUCK!

Exercise 1 (5 p)

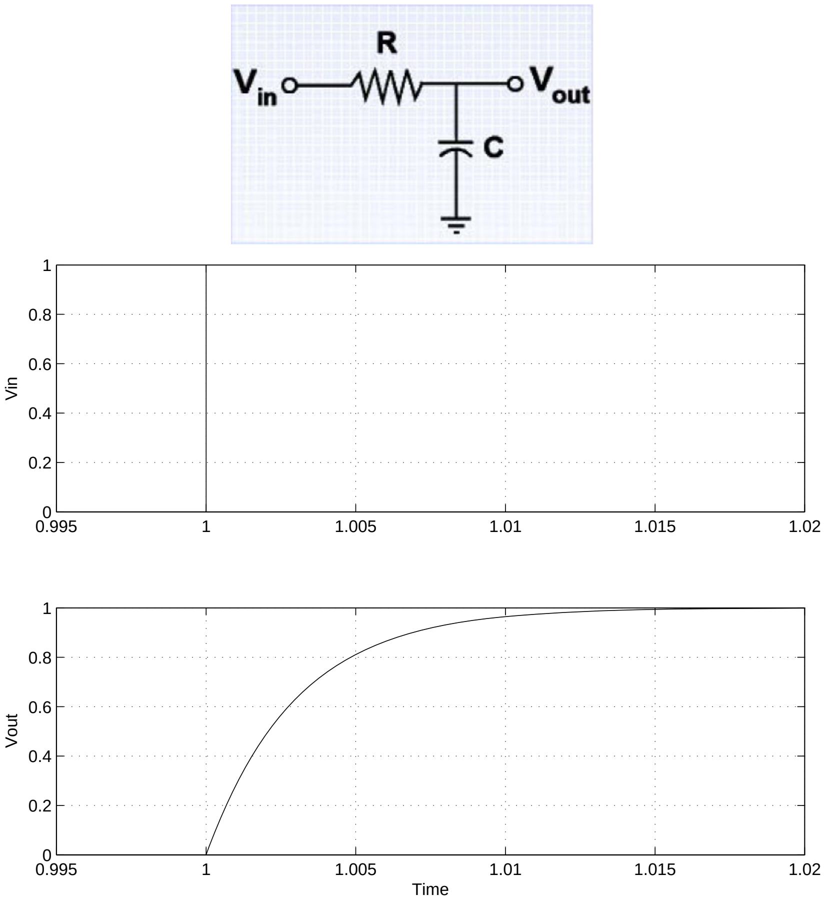
(a) Why do we build mathematical models? (Four reasons) (1p)

(b) Consider the second order system

$$\dot{x}_1 = x_2 + u$$
$$\dot{x}_2 = (1 - x_1^2)x_2 - x_1$$

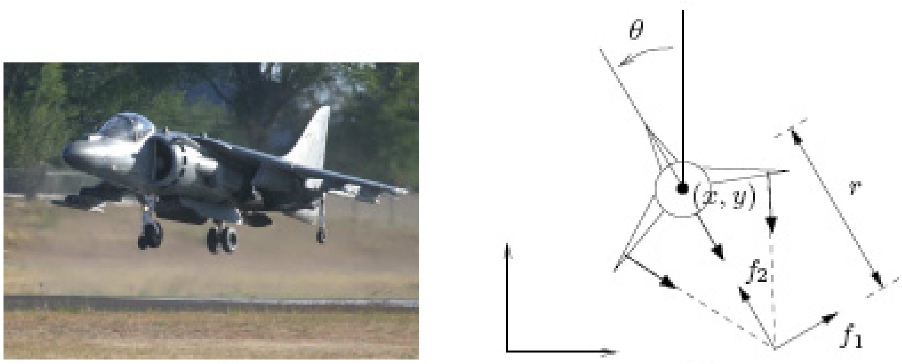
Determine an equilibrium point and linearize the system around that equilibrium point. (2p)

(c) In order to determine the value of a capacitor, you could couple it in series with a resistor with known resistance (R=1000 Ohm) and perform a step response. Determine the capacitor's value from the step response presented in the figure below! (2p)



Exercise 2 (5 p)

Consider the motion of vectored thrust aircraft, such as the Harrier ”jump jet” shown in the figure below.



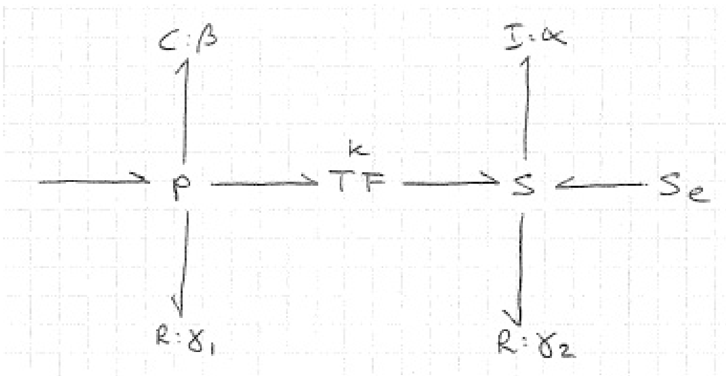
(a) (b)

The Harrier is capable of vertical takeoff by redirecting its thrust downward and through the use of smaller maneuvering thrusters located on its wings. A simplified model of the Harrier in a vertical plane through the wings of the aircraft is shown in the figure above. The forces generated by the main downward thruster and the maneuvering thrusters resolve as a pair of forces f_1 and f_2 acting at a distance r below the aircraft (determined by the geometry of the engines). Let (x, y, θ) denote the position and orientation of the center of mass of aircraft. Determine a state-space model (on the form $\dot{x} = f(x, u)$ for the Harrier aircraft!

Let m be the mass of the vehicle, J the moment of inertia and g the gravitational constant.

Exercise 3 (5 p)

A colleague of you have modelled a system using Bond graph theory. The bond graph looks like this:



Mark causality for the graph and determine whether the left hand side input should be an effort source or a flow source, if conflict free causality is desirable. Also determine a state space model for the system!

Exercise 4 (5 p)

Assume that we would like to estimate the parameters in the model

$$y(t) = b_1 u(t-1) + b_2 u(t-2) + e(t)$$

where $e(t)$ is white noise, using collected data.

(a) Show that the variance of the parameter estimates only depend on $R_u(0)$ and $R_u(1)$ where $R_u(0) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} u^2(t)$ and $R_u(1) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} u(t)u(t-1)$. What is the variance for $\hat{b}_1$ when $N \to \infty$ where N is the number of measurement points? Assume that the limit values $R_u(0)$ and $R_u(1)$ exists. (3p)

(b) Assume that $R_u(0) = 1$, i.e. that the variance for the input signal is given, but that $u(t)$ otherwise can be chosen freely. How should $u(t)$ be chosen in order to minimize the variance for $\hat{b}_1$? (2p)

Hints! The parameter estimate variance can be calculated as

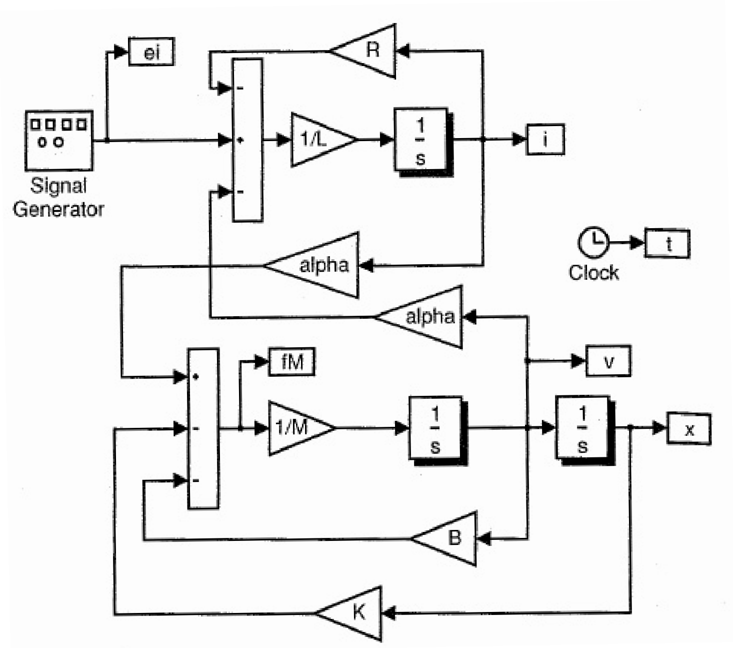
$$E\{(\theta_N - \theta_0)(\theta_N - \theta_0)^T\} = \frac{\lambda}{N}\bar{R}^{-1}$$

where

$$\bar{R} = E\{\psi(t)\psi^T(t)\} \qquad\qquad \psi(t) = \frac{d}{d\theta}\hat{y}(t|\theta)$$

Exercise 5 (5 p)

Consider the Simulink system below:



(a) Determine the underlying differential equations for the Simulink model above. (3p)

(b) The system modelled is a loudspeaker and the corresponding parameters are: L=0.02, K=1.25e5, R=2, B=30, M=0.002 and α=2. The eigenvalues of the system are:

```
>> eig(A)

ans =

  1.0e+004 *

   -0.5708
   -1.1157
   -9.8135
```

You would like to simulate the system using an Euler method, which is the longest step size that can be used in order to have a stable simulation? (2p)