ESS100 Modelling and simulation
Exam Mon, 18 December 2006

Exercise 1 (10 p)

(a) It is often useful to use variable step size when solving differential equations numerically (simulating). Why? (2p)

(b) How can the parameters a and b in the system

$$\frac{b}{s+a}$$

be estimated from a step response? (2p)

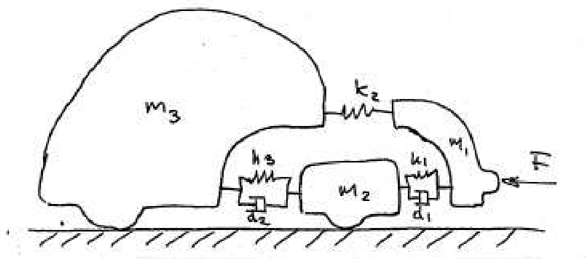
(c) Causality is a central concept when building models using bond graphs. What does it mean when a system is causal? (2p)

(d) What does model validation mean? Mention three different methods for model validation. (2p)

(e) How can you from step responses determine if a system is linear or nonlinear? (2p)

Exercise 2 (8 p)

The figure below shows how a vehicle can be described for crash safety. The beams in the vehicle are represented by springs and dampers in order to see how the vehicle is deformed during a crash.

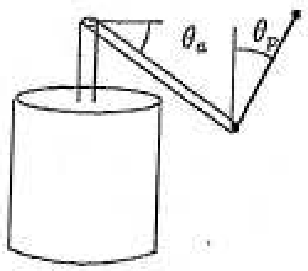


(a) Determine a bond graph for the vehicle and mark causality. Verify that the system has conflict free causality? (5p)

(b) Determine a state space representation for the vehicle. Let F be input. Write on the form $\dot{x} = Ax + Bu$. (3p)

Exercise 3 (6 p)

A simple inverted pendulum consists of a DC-motor with an arm mounted on the vertical axle. At the end of this arm is the pendulum attached, which is rotating perpendicular to the arm. See figure:



The pendulum is described by

$$\begin{aligned} J_p\ddot{\theta}_p &= mglsin\theta_p + \alpha cos\theta_p \\ J_a\ddot{\theta}_a &= \alpha M \end{aligned}$$

where M is the DC-motor torque. The DC-motor torque can be approximated as a first order system with time constant T and static gain K.

(a) Derive a state space model for the system on the form $\dot{x} = f(x, u)$. (4p)

(a) Linearize the system around a suitable operating point. (2p)

Exercise 4 (10 p)

We would like to use identification and the prediction error method to estimate the parameters in the model

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$

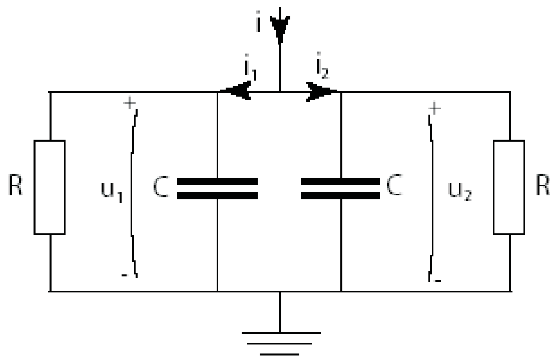
where $e(t)$ is white noise. The true system is

$$y(t) = 0.7u(t-1) + 0.3u(t-2) + v(t)$$

where the input, $u(t)$, is white noise with variance 1 and the disturbance, $v(t)$, is white noise with variance 2. v and u are assumed to be uncorrelated. To which values does the estimation of a and b converge, when the number of samples N is large ($N \to \infty$)?
Hint: Least squares estimation $\hat{\theta}_N$ can be calculated using

$$\hat{\theta}_N = (\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t))^{-1}(\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t))$$

Exercise 5 (8 p)



(a) Show that the system, with current i as input signal and the voltage $u = u1 = u2$ as output signal, can be described by a first order differential equation. (4p)

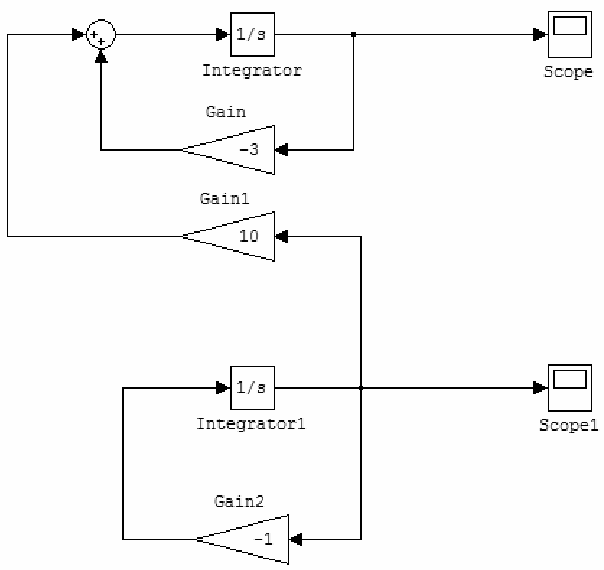
(b) The system can also be described using the following DAEs

$$\begin{aligned} C\tfrac{d}{dt}u_1 + \tfrac{1}{R}u_1 - i_1 &= 0 \\ C\tfrac{d}{dt}u_2 + \tfrac{1}{R}u_2 - i_2 &= 0 \\ u_1 - u_2 &= 0 \\ i_1 + i_2 - i &= 0 \end{aligned}$$

Determine index for the system. (4p)

Exercise 6 (8 p)

Consider the Simulink model.



(a) Determine a state space model for the system! (2p)

(b) You would like to simulate the system using an Euler method, which is the longest step size that can be used in order to have a stable simulation? (6p)