ESS100 Modelling and simulation
Solutions to exam Thu, 31 March 2005

Uppgift 1

(a) In a static system the output depends directly on the input (no memory), while in a dynamic system the output depend on old input signals and output signals.

(b) You get in general a better fit, smaller bias error, but the model becomes more sensitive to noise, since you model in the noise in the model (not only in the disturbance part).

(c) Aggregation of states, example difference approximation of the spacial variable.

(d) The system is on standard form:

$$\begin{bmatrix} I & 0 \\ 0 & N \end{bmatrix} \dot{x} + \begin{bmatrix} -A & 0 \\ 0 & I \end{bmatrix} x = \begin{bmatrix} B \\ D \end{bmatrix} u$$

The smallest integer k for which $N^k = 0$, is called the *index*. For our system *index*=3, because $N^1 \neq 0$ but $N^2 = 0$, but $N^3 = 0$.

(e) From the step response can the static gain and the time constant be determined. Static gain is given as b/a and the time constant as $1/a$.

Uppgift 2

(a) Introduce state variables, ex: torsion in the drive axles, $x_1 = \theta$, engine speed, $x_2 = \omega_{motor}$, and wheel speed, $x_3 = \omega_{hjul}$. The driveline model can now be written as

$$\begin{aligned} \dot{x}_1 &= x_2 - x_3/i \\ J_{motor}\dot{x}_2 &= T_{motor} - (kx_1 + dx_2 - dx_3/i)/i \\ J_{hjul}\dot{x}_3 &= (kx_1 + dx_2 - dx_3/i) - T_{load} \end{aligned}$$

where i is the gear ratio, J_{motor} and J_{hjul} are the engine's flywheel (inertia) and wheels inertia, d are the damping in the drive axles and k is its stiffness. The output signal is the wheel's longitudinal velocity which is given as the wheel speed times the wheel radius, $(v_{hjul} = r_{hjul}\omega_{hjul})$. This can now be written as:

$$\dot{x} = \begin{bmatrix} 0 & 1 & -1/i \\ -k/i/J_{motor} & -d/i/J_{motor} & d/i^2/J_{motor} \\ k/J_{hjul} & d/J_{hjul} & -d/i/J_{hjul} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/J_{motor} \\ 0 \end{bmatrix} T_{motor} + \begin{bmatrix} 0 \\ 0 \\ -1/J_{hjul} \end{bmatrix} T_{load}$$

$$y = \begin{bmatrix} 0 & 0 & r_{hjul} \end{bmatrix} x$$

(b) At low speed $v_{fordon} \approx 0$, slip is not defined - division by zero.

Uppgift 3

(a) Introduce the state variables $x_1 = y$ and $x_2 = \dot{y}$. The system can be written in state space form as

$$\begin{aligned} \dot{x}_1 &= x_2 && (= f_1(x_1, x_2, u)) \\ \dot{x}_2 &= -x_1 x_2 + 3u + 1 && (= f_2(x_1, x_2, u)) \end{aligned}$$

(b) In the stationary point it holds that

$$\dot{x}_1 = 0,\ \dot{x}_2 = 0,\ y_1 = x_{10} = 1$$

This gives

$$(x_{10}, x_{20}, u_0) = (1, 0, -1/3)$$

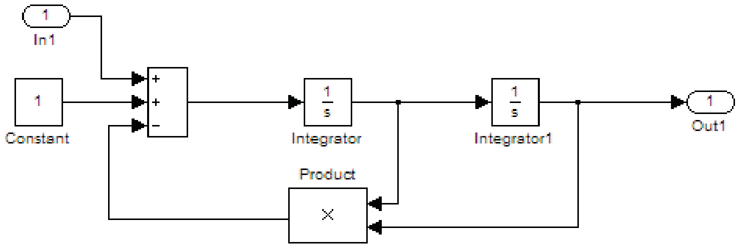
Linearize around the stationary point (x_{10}, x_{20}, u_0):

$$\Delta\dot{x} = \left.\frac{\partial f}{\partial x}\right|_{x_0,u_0} \Delta x + \left.\frac{\partial f}{\partial u}\right|_{x_0,u_0} \Delta u = A\Delta x + B\Delta u$$

where

$$\left.\frac{\partial f}{\partial x}\right|_{x_{10},x_{20},u_0} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad \left.\frac{\partial f}{\partial u}\right|_{x_{10},x_{20},u_0} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

(c)



Uppgift 4

The loss function $V(b)$ can be written as (when $N => \infty$)

$$\bar{V}(b) = E\{\varepsilon^2(t,\Theta)\} = E\{(y(t) - \hat{y}(t,\Theta))^2\} =$$

$$= E\{(0.6u(t-1) + 0.4u(t-2) + v(t) - bu(t-k))^2\} =$$

$$= (0.36{+}0.16{+}b^2)R_u(0){+}2{*}0.24{*}R_u(1){+}\lambda{-}2{*}0.6b{*}R_u(k{-}1){-}2{*}0.4bR_u(k{-}2)$$

Find minimum of $\bar{V}$ ($b^* = arg\ min \bar{V}(b)$):

$$\bar{V}'(b) = 2(bR_u(0) - 0.6R_u(k-1) - 0.4R_u(k-2)) = 0$$

$$=> \ b^* = (0.6R_u(k-1) + 0.4R_u(k-2))/R_u(0)$$

Check if this is minimum: $\bar{V}''(b) = 2R_u(0) > 0$

The estimation of b becomes:

$$b^* = \begin{cases} 0.6*1 + 0.4*0.5 = 0.8 & k = 1 \\ 0.6*0.5 + 0.4*1 = 0.7 & k = 2 \\ 0.6*0 + 0.4*0.5 = 0.2 & k = 3 \\ 0 & k > 3 \end{cases}$$

Uppgift 5

The system can be written as

$$w(t) = H(q)e(t) = \frac{q-0.4}{q-0.6}e(t)$$

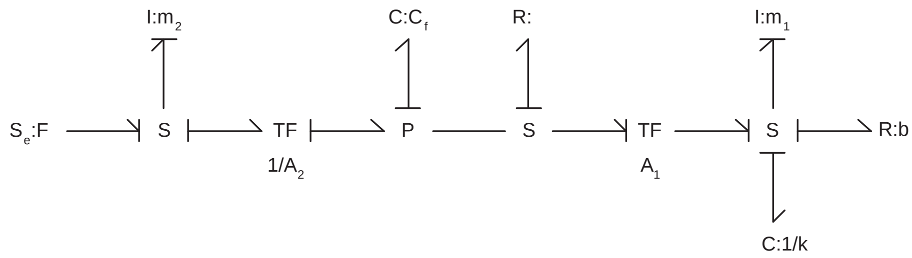
When $\Phi_e(\omega) = \lambda_e$ the spectrum of $\Phi_w(\omega)$ becomes

$$\Phi_w(\omega) = |H(q)|^2 \Phi_e(\omega) = \left| \frac{q - 0.4}{q - 0.6} \right|^2 \lambda_e = \frac{(e^{i\omega} - 0.4)(e^{-i\omega} - 0.4)}{(e^{i\omega} - 0.6)(e^{-i\omega} - 0.6)} \lambda_e =$$

$$= \frac{1.16 - 0.8cos(\omega)}{1.36 - 0.72cos(\omega)} \lambda_e$$

Uppgift 6

(a) Bond graph with no conflict in the causality rules:



$v(t)$ is to be input signal, otherwise conflicts in the bond graph.

(b) Input signal: $v(t)$
Output signal: $h(t)$
State variables: p (pressure in tank) or h (level in tank); v_1 (velocity for m_1); F_k (spring force) or x (position).