ESS100 Modelling and Simulation
Exam Tue, 14 December 2004

Exercise 1 (10 p)

(a) Mention one advantage and one disadvantage with implicit numeric methods. (2p)

(b) With an ARX-model it is simple to find an estimate of the parameters, since the problem becomes a linear regression problem and this can be solved effectively using least squares. Can the estimation of OE-models be solved using least squares? (Motivate) (2p)

(c) Determine index for the following DAE!

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} u$$

(2p)

(d) What does it mean when a system of differential equations is stiff? What difficulties appears when you try to simulate the system? (2p)

(e) Why do you linearize models? Which limitations does the linearized model have? (2p)

Exercise 2 (10 p)

A system

$$y(t) = G(p)u(t) + H(p)e(t)$$

is fed back via

$$u(t) = -F(p)y(t) + v(t)$$

where u and y are the systems input and output signals respectively. v and e are disturbances with known spectrum, Φ_v and Φ_e respectively.

To identify the closed loop system is spectral analysis used. In this way the estimation of $G(p)$ becomes

$$\hat{G}(i\omega) = \frac{\Phi_{yu}(\omega)}{\Phi_u(\omega)}$$

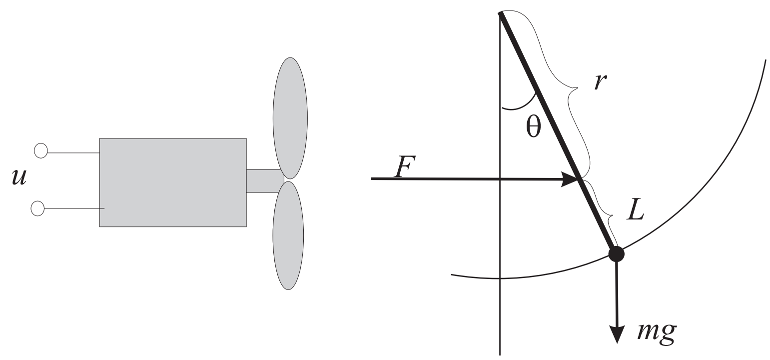
where Φ_{yu} is the cross spectrum between y and u, and Φ_u is the spectrum of u.

Determine Φ_{yu} and Φ_u and analyze and comment on the consequences for the estimation of $G(p)$ when the disturbance v disappears, $v(t) \approx 0$.

Hint: The spectrum for a signal is the absolute square of its fourier transform and the cross spectra between two signals y and u can be calculated as the product of the fourier transform of y and the complex conjugate if the fourier transform of u.

Exercise 3 (5 p)

The fan process used in the lab is to be studied.



Figur 1: Fan process.

The dynamics for the plate can be assumed to the descried as a pendulum:

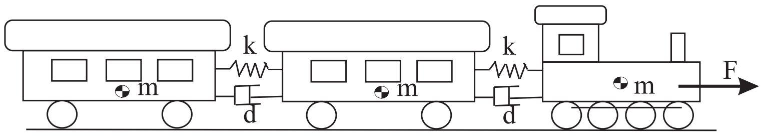
$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\tfrac{g}{r+L} sin x_1 - \tfrac{k}{m} x_2 + \tfrac{1}{m(r+L)^2} Fr cos x_1\end{aligned}$$

where x_1 is the angle θ, x_2 is the angular velocity $\dot{\theta}$. m, g, r and L are constants according to Figure 1, k is the friction at the hinge and F is the force the fan generates. The fan's force F can be controlled via the input signal u and the dynamics for the fan can be approximated as a first order system with static gain 1 and time constant T.

Augment the state space model with the dynamics for the fan and then linearize system around the operating point $(x_0 = 0, u_0 = 0)$.

Exercise 4 (15 p)

The figure below shows a train consisting of one locomotive and two wagons.



Figur 2: Train set.

(a) Draw a bond graph for the train and mark causality. Verify that the causality has no conflicts? (7p)

(b) Determine a state space model for the train. Let the locomotive's traction force be input signal and the distance between the locomotive and the last wagon be the output signal. The train has length L_0 at stand still (at rest). (It is possible to solve this part of the problem without solving the first part.) (7p)

(c) How would the state space representation look like if the train set consisted of n wagons instead of 2? (1p)

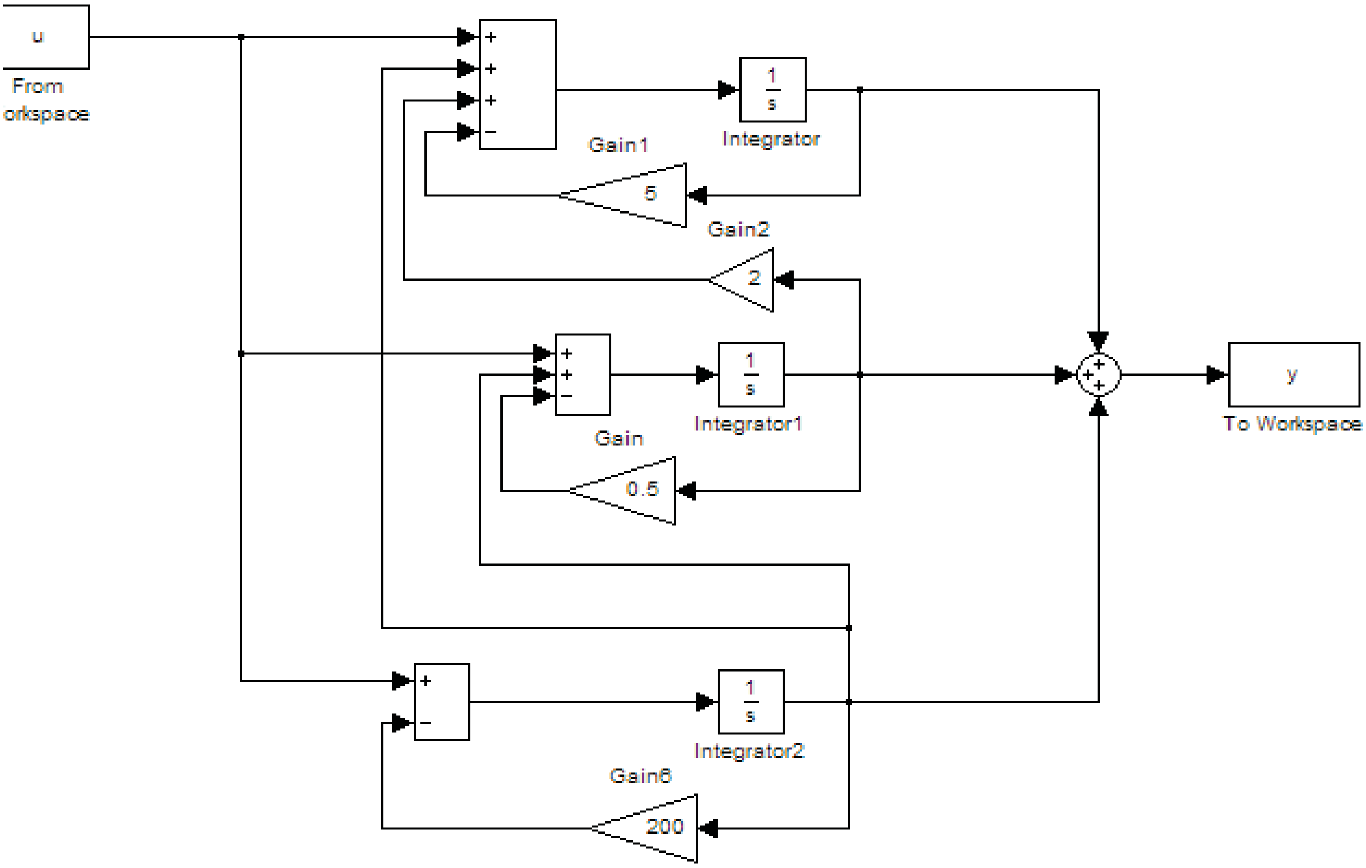
Exercise 5 (10 p)

Look at the Simulink scheme in Figure 3.

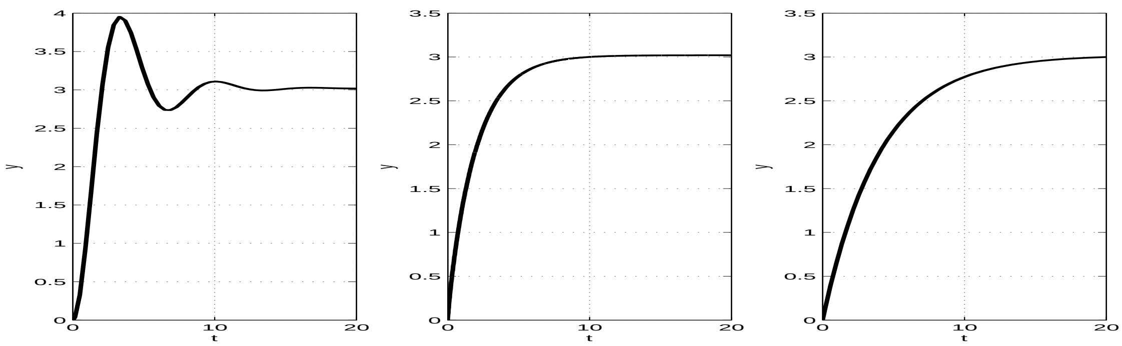
(a) Determine a state space model for the system. (5p)

(b) Figure 4 shows three different step responses. One belong to the Simulink model, which one? (Motivate) (2p)

(c) For control design, the model is a little bit to complex. Simplify the model by removing the fastest dynamics. (3p)



Figur 3: Simulink model.



Figur 4: Step response.