

Examination**ENM060 Power Electronic Converters****Date and time** Tuesday April 5th, 2016, 14:00 – 18:00**Responsible Teacher:** Andreas Henriksson, tel. 0709-524924**Authorised Aids:** Chalmers-approved calculator (Casio FX82..., Texas Instruments Ti-30... and Sharp EL-W531...)**Grades:** U, 3, 4 or 5. (The limit for a 3 on the exam is 20p, a 4 is 30p and 5 is 40p. The maximum number of points is 50.)**Solutions:** Course webpage (Ping-Pong), April 6th 2016**Review of Exam** May 2nd and May 4th, 12:00-13:00.
Fredrik Lamms Room. Division of Electric Power Engineering (1st floor).From May 5th 2016, the exams can be picked-up at the exam office, Department of Energy and Environment.

Location: EDIT building, Maskingränd 2, 3Ö (east) floor, room 3434A.

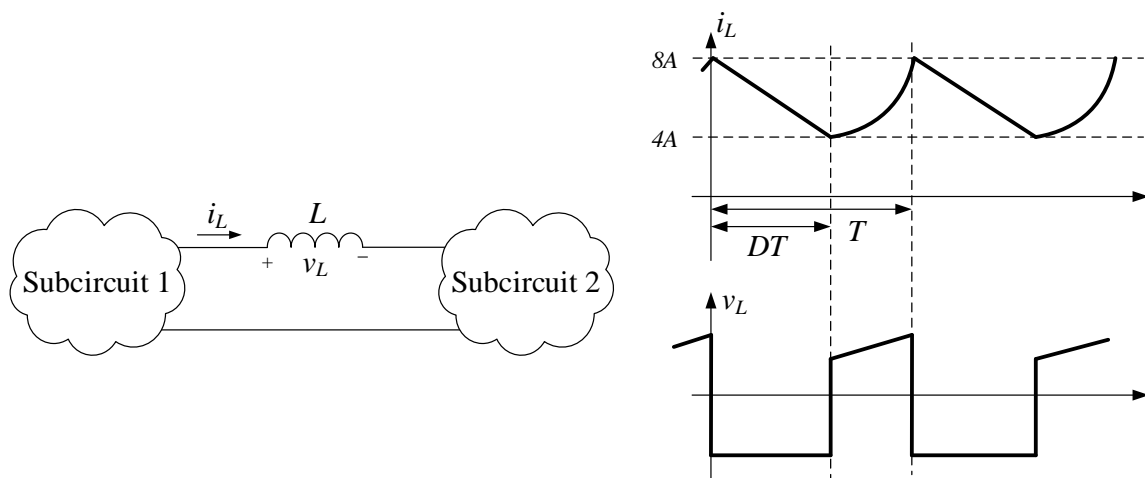
Opening hours during semesters: Monday-Friday 12:30-14:30

Observe that the questions are not arranged in any kind of order.

On the last pages there are some formulas that can be used in the examination. Always assume steady-state conditions in all tasks unless otherwise stated.

Please, read through the exam before you start.

- 1) For the circuit below with $T = 30\mu s$ and $D = 0.6$, sketch the resulting voltage and calculate the inductance needed if the voltage applied over the inductor during $0 < t < DT$ is 12V. (3p)

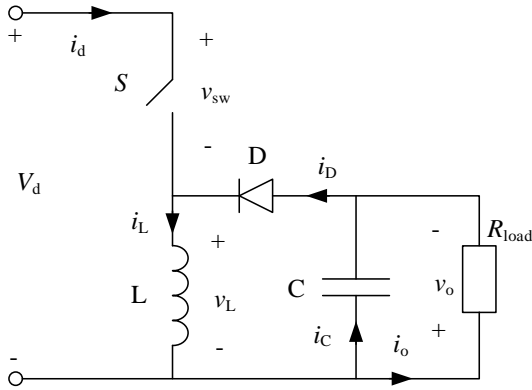
The constant inductor voltage during $DT < t < T$ is given. The resulting current through the inductor can be calculated as:

$$i_L = i_L(t_1) + \frac{1}{L} \int v_L dt$$

Which means that the current derivative will be positive when the applied voltage is positive.

$$L = \frac{\text{VoltSeconds}}{\Delta i_L} = \frac{0.6 \cdot 30\mu s \cdot 12V}{4A} = 54\mu H$$

- 2) For the buck/boost converter below, derive an expression for the ratio between the input and output voltage if the converter is operating in CCM. (3p)



For CCM we know that $\Delta i_L \leq 2I_L$

$$v_L = L \frac{di_L}{dt} \Rightarrow \Delta i_L = \frac{V_d D}{Lf_s}$$

For finding the input output voltage relation in CCM we calculate the average voltage over the inductor, which we know is zero in steady-state

$$0 = V_L = \frac{1}{T_s} (V_d D T_s - (1-D) T_s V_o) \Rightarrow \frac{V_o}{V_d} = \frac{D}{1-D}$$

- 3) The buck/boost converter in (2) is used with the circuit parameters and operating conditions below. Draw the resulting capacitor current (i_c), apply Simpsons formula and calculate the maximum power dissipation in the output capacitor if $R_{ESR} = 230m\Omega$. (5p)

$$V_d = 20V \quad V_o = 30V \quad L = 40\mu H \quad f_s = 80kHz \quad C_o = 330\mu F \quad I_o = 1A \text{ to } 7A$$

The input voltage can vary which gives a duty-cycle that also varies.

$$D = \frac{V_o}{V_o + V_d} \rightarrow D = \frac{30V}{30V + 20V} = 0.60$$

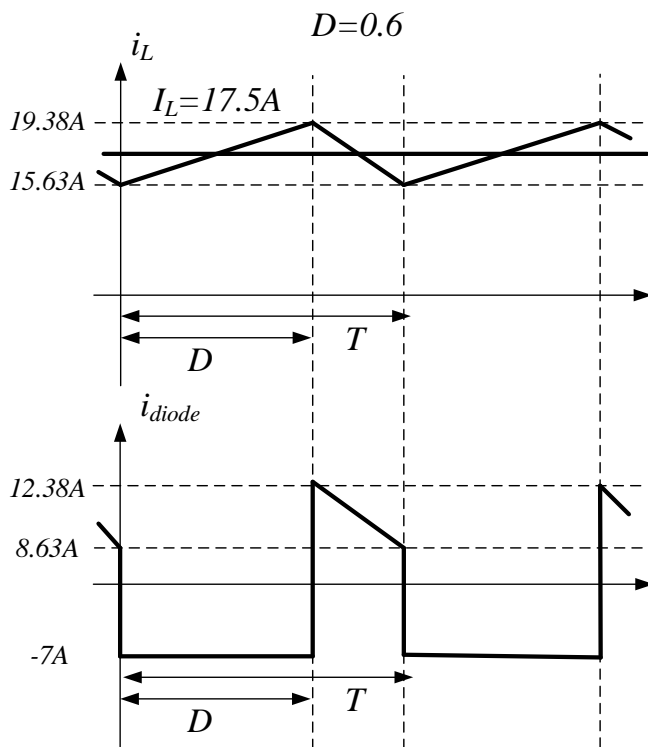
To find I_L we calculate the average output current

$$I_o = \frac{V_o}{R_{load}} = \frac{1}{T_s} \int_0^{T_s} i_o(t) dt = \frac{1}{T_s} \int_0^{T_s} i_D(t) dt - \underbrace{\frac{1}{T_s} \int_0^{T_s} i_c(t) dt}_{=0} = I_D \Rightarrow$$

$$I_o = \frac{V_o}{R_{load}} = \frac{1}{T_s} \int_0^{T_s} i_o(t) dt = (1-D) \underbrace{\frac{1}{(1-D)T_s} \int_{DT_s}^{T_s} i_L(t) dt}_{=I_L} = (1-D)I_L$$

$$I_L = \frac{I_o}{1-D} = \frac{7A}{1-0.6} = 17.5A$$

$$v_L = L \frac{di_L}{dt} = L \frac{\Delta i_L}{\Delta t} \quad \rightarrow \quad \Delta i_L = \frac{V_a D T_s}{L} = \frac{20V \cdot 0.6}{40\mu H \cdot 80kHz} = 3.75A$$



To calculate the power loss in the capacitor, only the equivalent series resistance (R_{ESR}) is accounted for. This gives a need to calculate the RMS-current through the capacitor. The definition of Simpsons rule is:

$$\frac{1}{T} \int_{t_0}^{t_0+T} f(x) dx = \frac{1}{6} \left(f(t_0) + 4 \cdot f\left(t_0 + \frac{T}{2}\right) + f(t_0 + T) \right)$$

The RMS current can now be calculated as:

$$I_{C(RMS)} = \sqrt{\frac{1}{12.5\mu} \left(7.5\mu \int_0^{7.5\mu} (-7A)^2 dt + \frac{5\mu}{5\mu} \int_{7.5\mu}^{12.5\mu} i_C(t) dt \right)}$$

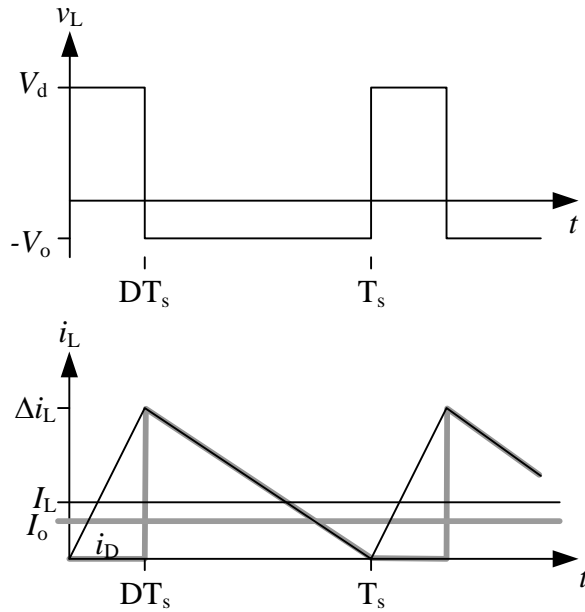
$$= \sqrt{\frac{7.5\mu}{12.5\mu} (-7A)^2 + \frac{5\mu}{12.5\mu} \frac{1}{6} ((12.38A)^2 + 4(10.5A)^2 + (8.63A)^2)} = 8.60A$$

The resulting power dissipation in the capacitor can now be calculated as:

$$P_C = 8.60A^2 \cdot 45m\Omega = 3.33W$$

- 4) **For the buck/boost converter in (2), calculate the lowest value of the inductance (L) required to keep the converter operation in continuous conduction mode (CCM) for the entire operating region. (4p)**

Start by calculating the border between CCM and DCM to find out which mode the converter is operating in. We start by drawing the inductor voltage and the inductor current at the border between CCM and DCM.



For CCM we know that $\Delta i_L \leq 2I_L$

$$v_L = L \frac{di_L}{dt} \Rightarrow \Delta i_L = \frac{V_d D}{Lf_s}$$

For finding the input output voltage relation in CCM we calculate the average voltage over the inductor, which we know is zero in steady-state

$$0 = V_L = \frac{1}{T_s} (V_d DT_s - (1-D)T_s V_o) \Rightarrow V_o = \frac{D}{1-D} V_d$$

To find I_L we calculate the average output current

$$I_o = \frac{V_o}{R_{load}} = \frac{1}{T_s} \int_0^{T_s} i_o(t) dt = \frac{1}{T_s} \int_0^{T_s} i_D(t) - i_C(t) dt = \frac{1}{T_s} \int_0^{T_s} i_D(t) dt - \underbrace{\frac{1}{T_s} \int_0^{T_s} i_C(t) dt}_{=0} = I_D \Rightarrow$$

$$I_o = \frac{V_o}{R_{load}} = \frac{1}{T_s} \int_0^{T_s} i_o(t) dt = (1-D) \underbrace{\frac{1}{(1-D)T_s} \int_{DT_s}^{T_s} i_L(t) dt}_{=I_L} = (1-D)I_L$$

For CCM we know

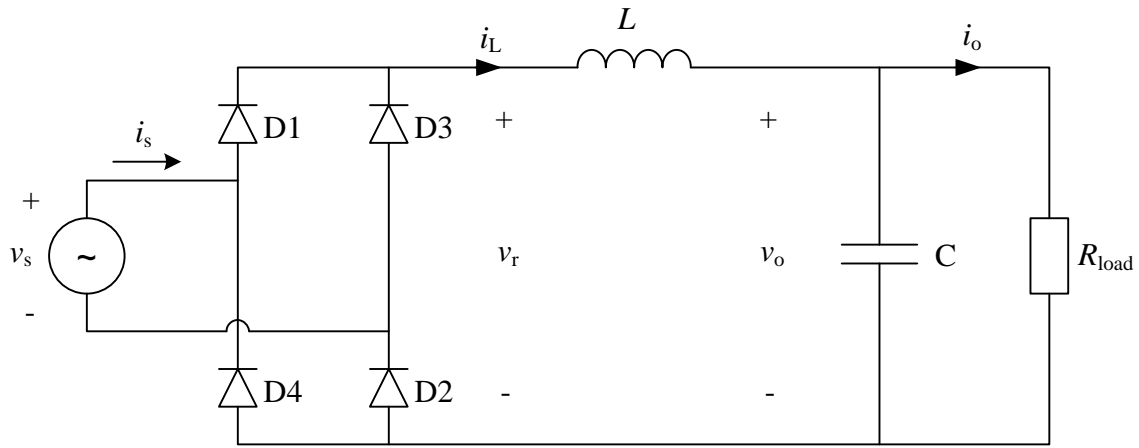
$$\Delta i_L \leq 2I_L \quad \Delta i_L = \frac{V_d D}{Lf_s} \quad I_L = \frac{I_o}{(1-D)} = \frac{V_o}{R_{load}} \frac{1}{(1-D)} \Rightarrow$$

$$\frac{V_d D}{Lf_s} \leq 2 \frac{V_o}{R_{load}} \frac{1}{1-D} \Rightarrow L \leq \frac{V_d R_{load} D(1-D)}{2V_o f_s} = \left[\frac{V_d D}{V_o} = 1-D \right] = R_{load} \frac{(1-D)^2}{2f_s}$$

From this we can see that the lowest output current gives the smallest inductance that will give CCM operation in the whole operating region. We also remember that $D=0.6$ in CCM

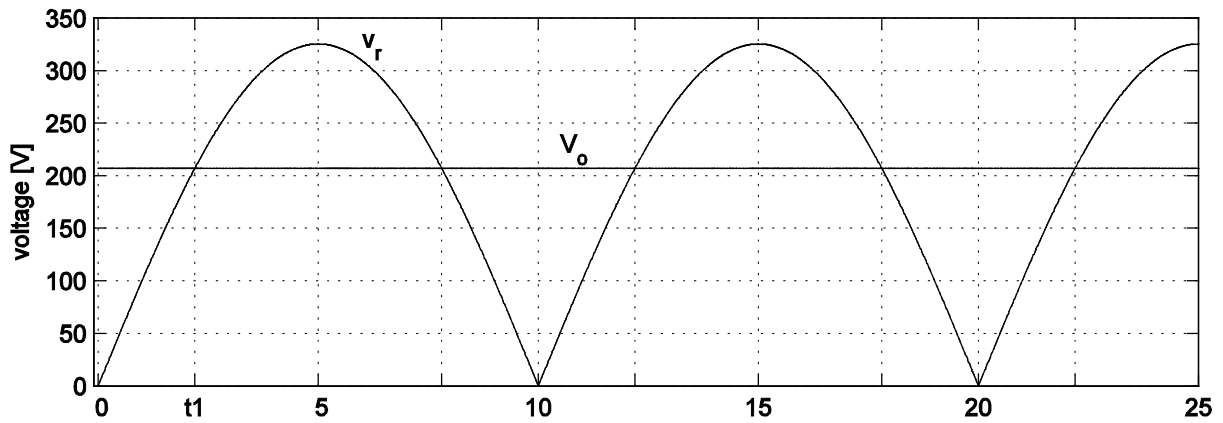
$$L_{min} = \frac{V_o (1-D)^2}{I_o 2f_s} = \frac{30V (1-0.6)^2}{1A \cdot 2 \cdot 80kHz} = 30\mu H$$

- 5) **The single-phase diode rectifier shown below is connected to an AC-voltage of 230V, 50Hz with a negligible inductance (source inductance). A DC-side inductance (L) is connected between the rectifier output and the filter capacitor (C). The DC-side inductance can not be considered infinite ($L=10mH$) but the filter capacitor can be assumed to be very large. Calculate the output voltage for a continuous inductor current (i_L). (3p)**



Assume steady-state operation and a constant DC-voltage since C is large. We start by drawing the voltage waveforms, since i_L is continuous, v_r will be equal to the rectified AC-voltage

$$v_r(t) = |\sqrt{2} \cdot 230V \cdot \sin(\omega t)|$$

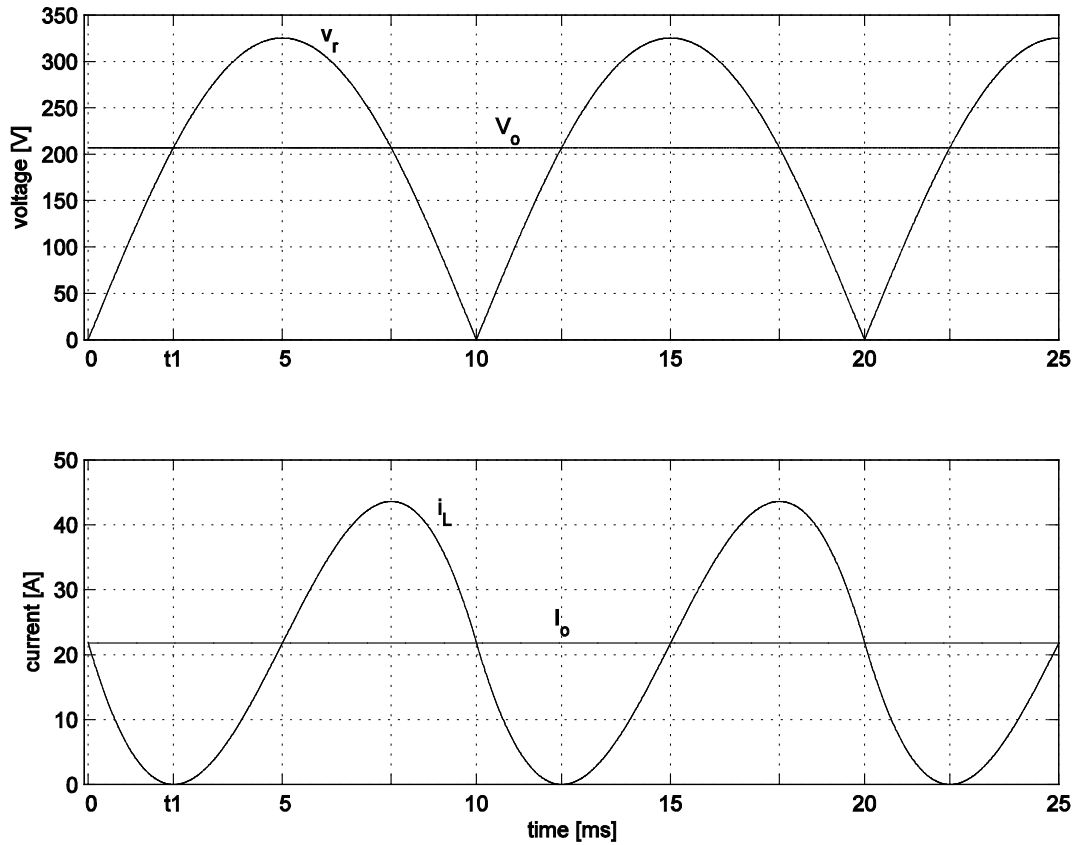


The average voltage across the inductor (L) must be zero in steady-state. This gives that the output voltage must be equal to the average of the rectified voltage. The average value of the capacitor voltage can then be computed as:

$$V_o = \frac{1}{T/2} \int_0^{T/2} v_r(t) dt = \frac{1}{20ms/2} \int_0^{20ms/2} \sqrt{2} \cdot 230V \cdot \sin(\omega t) dt = 207V$$

- 6) For the diode rectifier in (5), the current that flows through the DC-side inductance (i_L) will consist of a DC-component and a superimposed AC-component. Draw the waveforms of $v_r(t)$, $v_o(t)$ and $i_L(t)$ at the limit which $i_L(t)$ becomes discontinuous. No calculations are needed, just typical waveforms. (3p)

The current should increase when $v_r > V_o$ and decrease when $v_r < V_o$. The current should be zero when the rectified voltage increases and reach the output voltage and it should have its peak value when the rectified voltage decreases and reaches the output voltage.



- 7) For the diode rectifier in (5), the current that flows through the DC-side inductance (i_L) will consist of a DC-component and a superimposed AC-component. As the load resistance decreases or increases, the current can be either continuous or discontinuous. Calculate the load resistance value (R_{load}) at the limit below which $i_L(t)$ becomes discontinuous. (5p)

Since the output voltage (V_o) is constant, the load resistance is simply obtained from Ohms law and by using the average inductor current. The average inductor current can be calculated as follows:

$$i_L(t) = i_L(0) + \frac{1}{L} \int_0^t (v_R(t') - V_o) dt' = i_L(0) + \frac{1}{L} \int_0^t (\sqrt{2}V_{s,rms} \sin(\omega t') - V_o) dt'$$

$$= i_L(0) + \frac{1}{L} \left(\frac{\sqrt{2}V_{s,rms} (1 - \cos(\omega t))}{\omega} - V_o t \right).$$

At $t = t_1$, we have

$$\sqrt{2}V_{s,rms} \sin(\omega t_1) = V_o \Rightarrow t_1 = \frac{\arcsin\left(\frac{V_o}{\sqrt{2}V_{s,rms}}\right)}{\omega} = \frac{\arcsin\left(\frac{207}{\sqrt{2} \cdot 230}\right)}{2\pi \cdot 50} \approx 2.2 \text{ ms.}$$

At $t = t_1$, we also have

$$i_L(t_1) = i_L(0) + \frac{1}{L} \left(\frac{\sqrt{2}V_{s,rms} (1 - \cos(\omega t_1))}{\omega} - V_o t_1 \right) = 0$$

$$\Rightarrow i_L(0) = \frac{-\sqrt{2}V_{s,rms} + \omega V_o t_1 + \sqrt{2}V_{s,rms} \cos(\omega t_1)}{\omega L} \approx 21.8 \text{ A.}$$

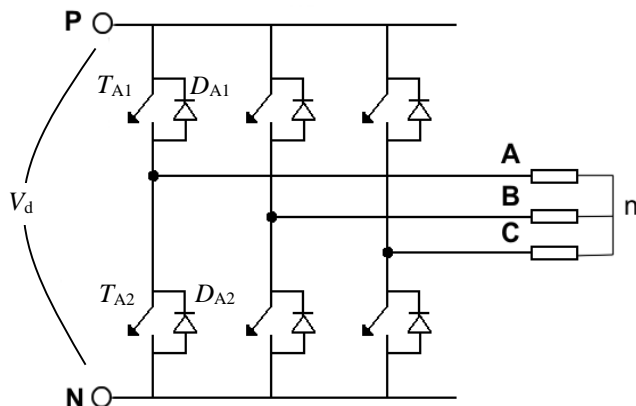
Now, the average inductor current can be calculated as

$$I_o = \frac{1}{T/2} \int_0^{T/2} i_L(t) dt = i_L(0) + \frac{2}{TL} \int_0^{T/2} \left(\frac{\sqrt{2}V_{s,rms}(1 - \cos(\omega t))}{\omega} - V_o t \right) dt = i_L(0) + 0 = i_L(0) = 21.8 \text{ A}$$

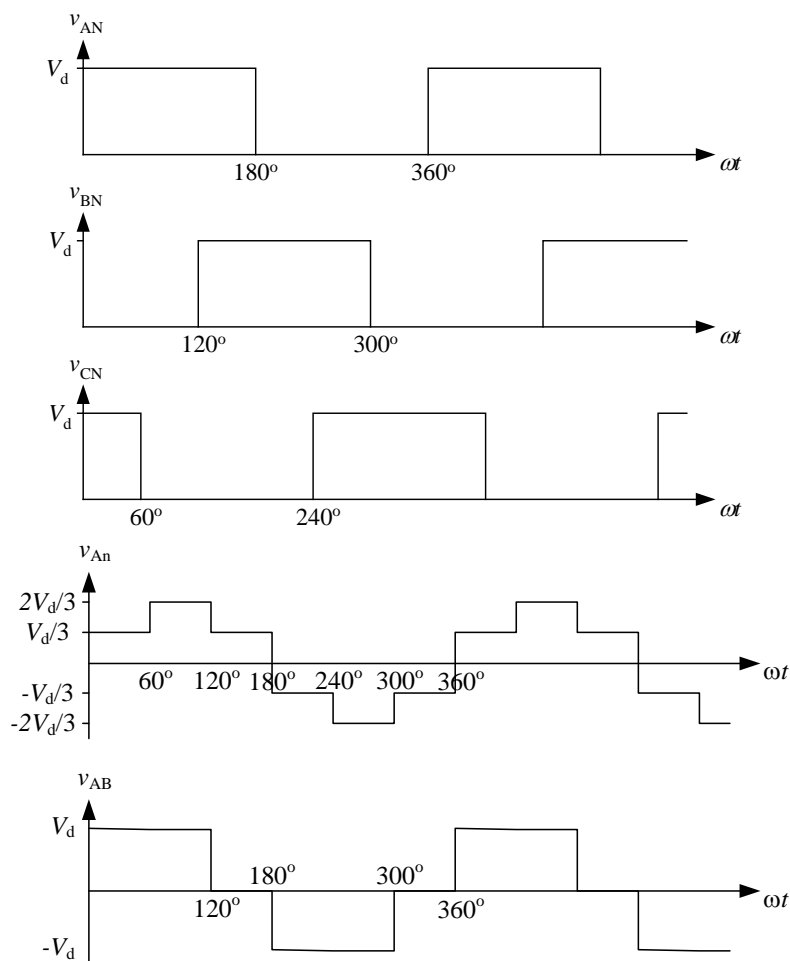
Hence, the resistance is now found as

$$R_{load} = V_o / I_o = 207V / 21.8A = 9.5\Omega$$

8) Consider the three phase inverter below.



The inverter is operated in square wave mode and the fundamental frequency component of the AC-side voltage is 50Hz. Plot the phase potentials of each leg (v_{AN} , v_{BN} , v_{CN}), the phase voltage over the load (v_{An}) and the resulting line to line voltage (v_{AB}). (4p)

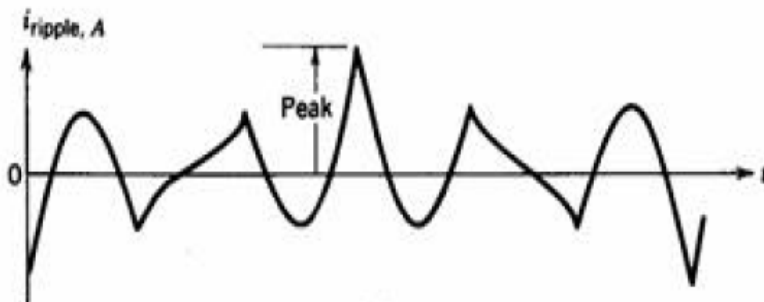


- 9) The three-phase inverter in (8) is loaded with a purely resistive load ($R_A = R_B = R_C = 5\Omega$) and with $V_d = 340V$. Determine the RMS-value of the fundamental component of the phase current (i_A). (4p)

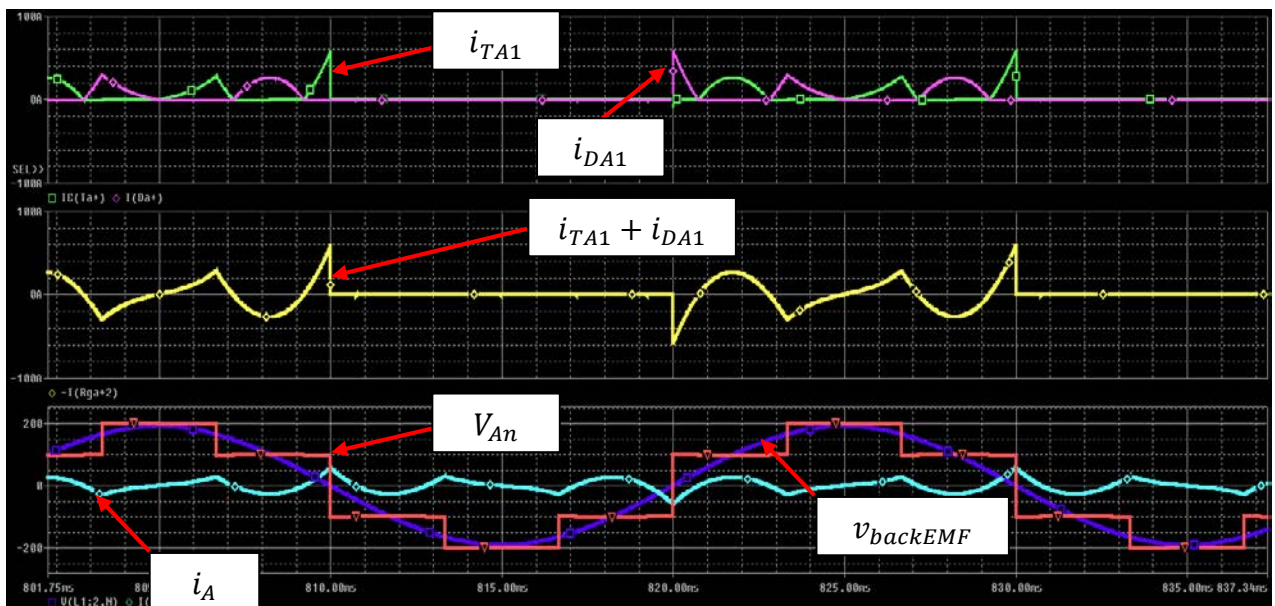
In square-wave operating mode, the output phase current will be a quasi-square wave shaped waveform. The fundamental component of the current (odd and half-wave) is calculated as:

$$\begin{aligned}
 b_n &= \frac{4}{\pi} \int_0^{\pi/2} f(t) \sin(n\omega t) d\omega t = \frac{4}{\pi} \int_0^{\pi/3} \frac{V_d}{3 \cdot 5\Omega} \cdot \sin(n\omega t) d\omega t + \frac{4}{\pi} \int_{\pi/3}^{\pi/2} \frac{2 \cdot V_d}{3 \cdot 5\Omega} \cdot \sin(n\omega t) d\omega t \\
 &= \frac{-4 \cdot V_d}{3\pi \cdot 5\Omega} [\cos(n\omega t)]_0^{\pi/3} - \frac{8 \cdot V_d}{3 \cdot 5\Omega} [\cos(n\omega t)]_{\pi/3}^{\pi/2} = \frac{-4 \cdot V_d}{3\pi \cdot 5\Omega} \left[\underbrace{\cos(\pi/3)}_{0.5} - \underbrace{\cos(0)}_{=1} \right] - \frac{8 \cdot V_d}{3\pi \cdot 5\Omega} \left[\underbrace{\cos(\pi/2)}_0 - \underbrace{\cos(\pi/3)}_{0.5} \right] \\
 &= \frac{-4 \cdot V_d}{3\pi \cdot 5\Omega} \left[\underbrace{\cos(\pi/3)}_{0.5} - \underbrace{\cos(0)}_{=1} \right] - \frac{8 \cdot V_d}{3\pi \cdot 5\Omega} \left[\underbrace{\cos(\pi/2)}_0 - \underbrace{\cos(\pi/3)}_{0.5} \right] = \frac{2 \cdot 340V}{\pi \cdot 5\Omega} \rightarrow i_{A(RMS)} = \frac{2 \cdot 340V}{\sqrt{2} \cdot \pi \cdot 5\Omega} = 30.6A
 \end{aligned}$$

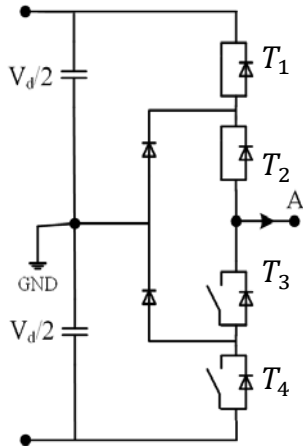
- 10) The load is replaced with an inductance and a voltage source (back-EMF) instead. The voltage from the back-EMF is sinusoidal shaped and in phase with the fundamental frequency output voltage which gives a resulting current as depicted below. For one leg and one period, sketch the phase voltage (v_{An}) and the back-EMF. Also, draw the current through phase leg A and the current through each diode (D_{A1}, D_{A2}) and each valve (T_{A1}, T_{A2}) (5p)



The example presented in the demonstrations calculates the ripple current when the converter is operating in square wave mode. It should therefore be able to draw the conclusion that the inverter is operating in square wave operating mode by looking at the give figure of the output phase current.



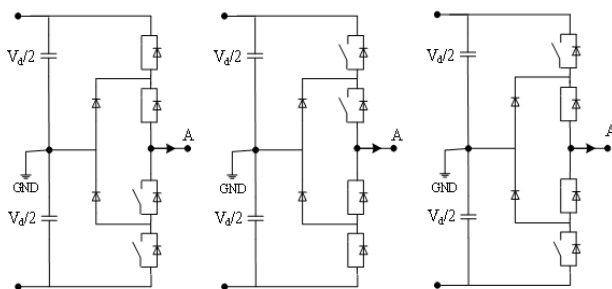
- 11) You replace the standard 2-level inverter in (8) with a 3-level neutral point diode clamped (NPC) inverter instead, see phase leg below. Draw the resulting mid-point voltages ($v_{AGND}, v_{BGND}, v_{CGND}$) and the resulting voltage (v_{An}) that is applied over a y-connected load (phase voltage) for the 3-level inverter. Assume that upper and lower part of the phase leg (T_1, T_2 or T_3, T_4) is turned on for 150° and that the midpoint switches (T_2, T_3) are turned on for 30° . What is the benefit of using a multi-level inverter? (4p)



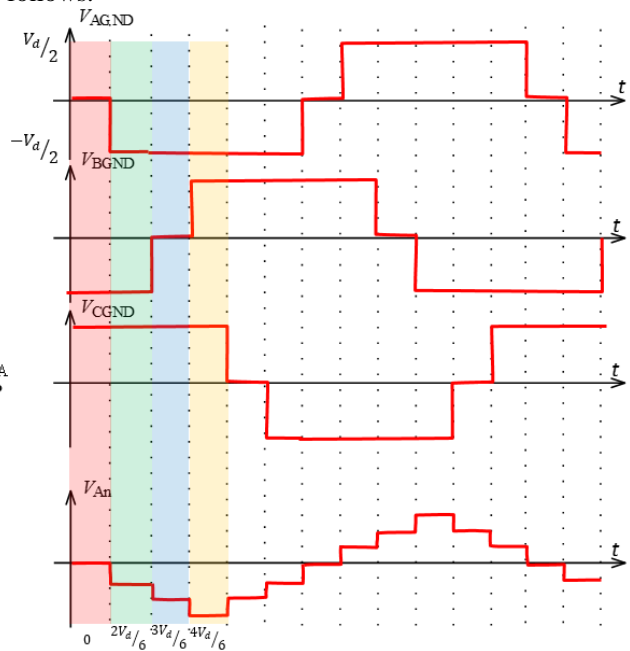
$$v_{An} = \frac{2}{3}v_{AGND} - \frac{1}{3}(v_{BGND} + v_{CGND})$$

The resulting voltage for the NPC-inverter can be drawn as follows:

- Bus-splitting common DC-capacitor and diodes for clamping
- The resulting phase voltage will have 4 levels



$$v_{An} = \frac{2}{3}v_{AGND} - \frac{1}{3}(v_{BGND} + v_{CGND})$$



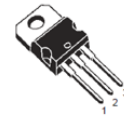
As seen from the graphs, the multi-level inverter gives a phase voltage with more steps, hence will the resulting harmonic content be lower.

- 12) For the component below, calculate the resulting chip temperature if the component is mounted on a heat sink with $R_{\theta SA} = 12.3K/W$, the power dissipation is $3.4W$ and the ambient temperature is $T_A = 25^\circ C$. If the component is used in an environment where it is exposed to many thermal cycles it will eventually break down. Due to which mechanism will it break down? (3p)

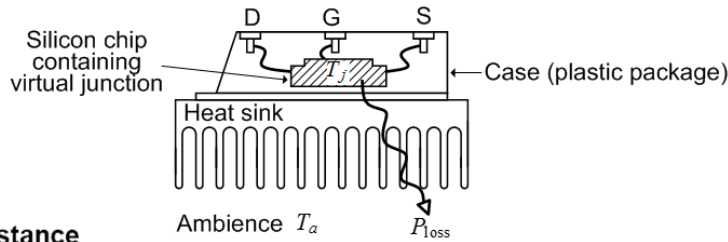


BUZ10

N - CHANNEL 50V - 0.06Ω - 23A TO-220
STripFET™ MOSFET



TO-220



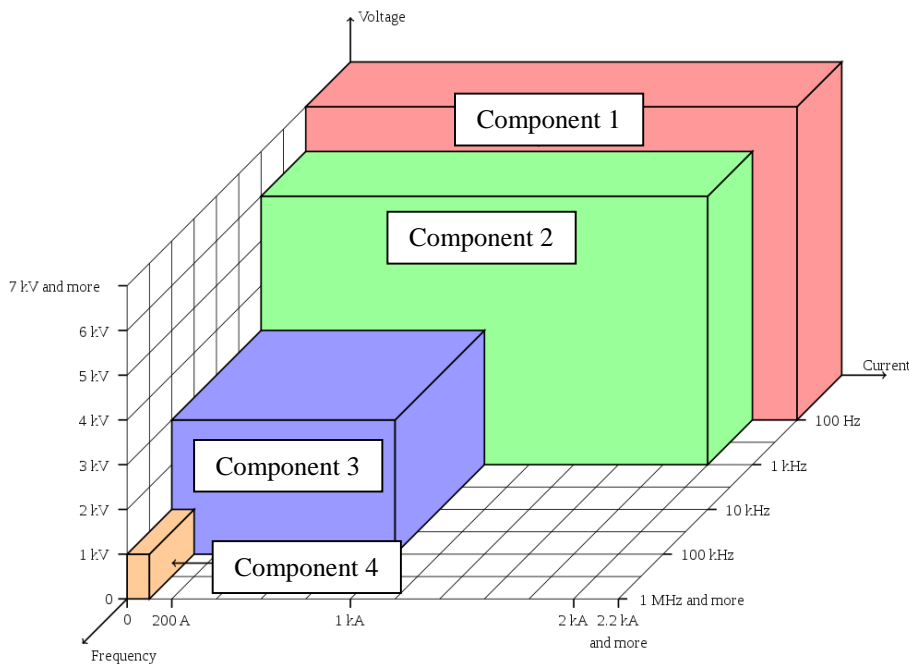
Thermal Resistance

	Parameter	Typ.	Max.	Units
$R_{\theta JC}$	Junction-to-Case	—	1.0	°C/W
$R_{\theta CS}$	Case-to-Sink, Flat, Greased Surface ④	0.50	—	
$R_{\theta JA}$	Junction-to-Ambient④	—	62	
$R_{\theta JA}$	Junction-to-Ambient (PCB mount)⑤	—	40	

$$T_j = (R_{\theta JC} + R_{\theta CS} + R_{\theta SA}) \cdot 3.4W + T_A = (1.0K/W + 0.5K/W + 12.3K/W) \cdot 3.4W + 25^\circ C = 72^\circ C$$

If the component is exposed to many thermal cycles it will eventually break down due to mechanical stress on the packaging (e.g. bond wires and die attachment).

- 13) In the diagram below, state which component that is typically used in each power range. Why can't high power components be used with a high switching frequency? (4p)



- Component 1 – Thyristor
- Component 2 – GTO
- Component 3 – IGBT
- Component 4 – MOSFET

High power components are typically optimized to have low conduction losses. As a consequence of this optimization and the high current/voltage levels, the switching losses will be high if used with a high switching frequency.

Formulas for Examination in Power Electronic Converters (ENM060)

Table 3-1 Use of Symmetry in Fourier Analysis

Symmetry	Condition Required	a_h and b_h	
Even	$f(-t) = f(t)$	$b_h = 0$	$a_h = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(h\omega t) d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0$	$b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even h $a_h = \frac{2}{\pi} \int_0^{\pi} f(t) \cos(h\omega t) d(\omega t)$ for odd h $b_h = \frac{2}{\pi} \int_0^{\pi} f(t) \sin(h\omega t) d(\omega t)$ for odd h	
Even quarter-wave	Even and half-wave	$b_h = 0$ for all h	$a_h = \begin{cases} \frac{4}{\pi} \int_0^{\pi/2} f(t) \cos(h\omega t) d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all h	$b_h = \begin{cases} \frac{4}{\pi} \int_0^{\pi/2} f(t) \sin(h\omega t) d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$

Definition of RMS-value:

$$F_{RMS} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} f(t)^2 dt}$$

Definition of RMS-value with Fourier-series:

$$F_{RMS} = \sqrt{F_0^2 + \sum_{n=1}^{\infty} F_n^2} = \sqrt{\left(\frac{\alpha_0}{2}\right)^2 + \sum_{n=1}^{\infty} \left(\frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}\right)^2}$$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\sin(\alpha) \sin(\beta) = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos(\alpha) \cos(\beta) = \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax), \quad \int x \sin(ax) dx = \frac{1}{a^2} (\sin(ax) - ax \cos(ax)), \quad \int \cos(ax) dx = \frac{1}{a} \sin(ax)$$

$$\int x \cos(ax) dx = \frac{1}{a^2} (\cos(ax) + ax \sin(ax))$$

$$PF = \frac{P}{S} = \frac{V_s I_{s1} \cos \phi_1}{V_s I_s}, \quad DPF = \cos \phi_1, \quad \%THD_i = 100 \frac{I_{dis}}{I_{s1}} = 100 \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = 100 \sqrt{\sum_{h \neq 1} \left(\frac{I_{sh}}{I_{s1}}\right)^2}$$

Electromagnetics

$$e = \frac{d}{dt} \psi \quad \psi = N\phi \quad \phi = BA \quad R = \frac{l}{A\mu_r\mu_0} \quad L = \frac{\Psi}{i}$$

$$NI = R\phi = mmf \quad N\phi = LI \quad L = A_L N^2 \quad W = \frac{1}{2} LI^2$$

Simpson's rule

Let $f(x)$ be a polynomial of maximum third degree, this means

$$f(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

For this function the integral can be calculated as

$$\frac{1}{T} \int_{t_0}^{t_0+T} f(x) dx = \frac{1}{6} \left(f(t_0) + 4f\left(t_0 + \frac{T}{2}\right) + f(t_0 + T) \right)$$