Examination **ENM060 Power Electronic Converters**

Date and time Monday January 12^{th}, 2015, 14:00 – 18:00

Responsible Teacher: Andreas Karvonen, tel. 0709-524924

Authorised Aids: Chalmers-approved calculator (Casio FX82..., Texas Instruments Ti30... and Sharp ELW531...)

Grades: U, 3, 4 or 5. (The limit for a 3 on the exam is 20p, a 4 is 30p and 5 is 40p. The maximum number of points is 50.)

Solutions: Course webpage (Ping-Pong), January 13^{th} 2015

Review of Exam February 5^{th} and February 11^{th}, 12:00-13:00.
Uno Lamms Room. Division of Electric Power Engineering (2^{nd} floor).

From February 12^{th} 2015, the exams can be picked-up at the student office at the department of Energy and Environment.
Location: EDIT building, Maskingränd 2, 3Ö (east) floor, room 3434A.
Opening hours during semesters: Monday-Friday 12:30-14:30

Observe that the questions are not arranged in any kind of order.

On the last pages there are some formulas that can be used in the examination. Always assume steady-state conditions in all tasks unless otherwise stated.

Please, read through the exam before you start.

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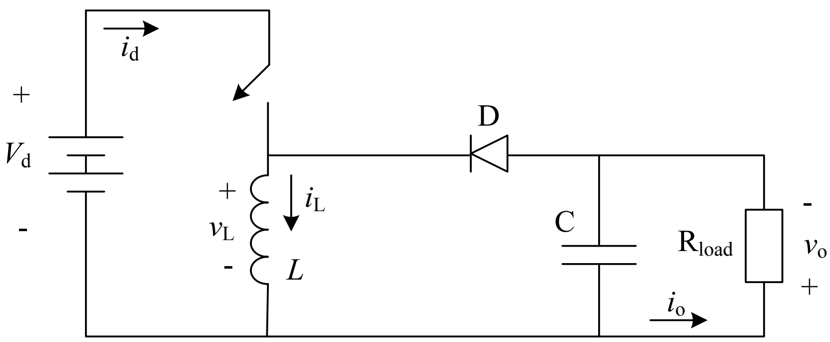
1) Consider the ideal buck/boost converter below. Derive the input/output voltage ratio for CCM. A complete derivation is needed; just writing down the final expression gives zero points. (2p)



$V_d = 9V$ $\quad V_o = 24V$ $\quad f_{sw} = 100kHz$ $\quad C = 470\mu F$ $\quad I_o = 2.5A$ $\quad L = 63\mu H$

$$V_L = \frac{1}{T_{sw}}\int_0^{DT_{sw}} V_d dt + \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} -V_o dt = \frac{1}{T_{sw}}(V_d DT_{sw} - V_o T_{sw} + V_o DT_{sw}) = 0$$

$$V_d D = V_o(1-D) \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D}{1-D}$$

2) For the buck/boost converter in question (1), an LC-filter is applied to the input that smoothens the current drawn from the DC-source. Calculate the power dissipation in the input filter capacitor if $R_{ESR} = 120m\Omega$. Assume that the input filter is sufficiently large so that the current from the DC-source is a pure DC-current. (5p)

Since the output voltage is held constant at 24V, the switch duty ratio D can be calculated:

$$V_d D = V_o(1-D) \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D}{1-D} \quad \rightarrow \quad \frac{24V}{9V} = \frac{D}{1-D} \quad \rightarrow \quad D = 0.727$$

During the on-time of the switch, the inductor voltage is constant $(v_L = V_d)$ which gives

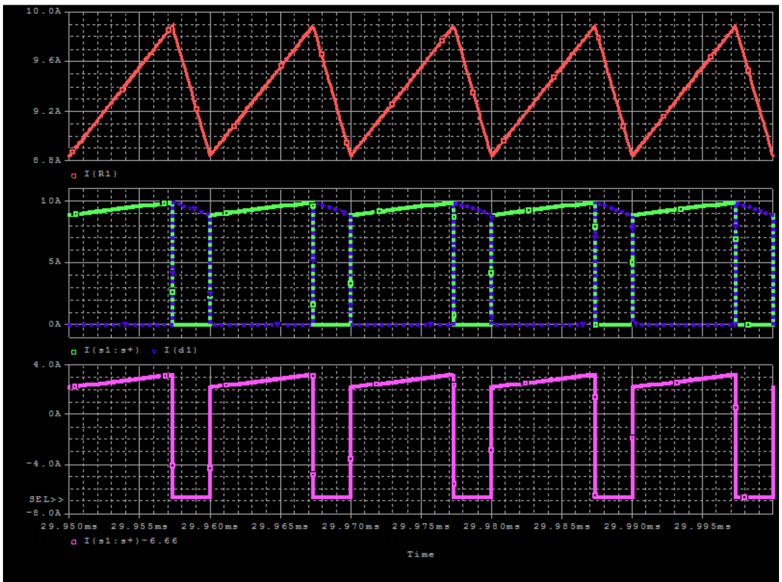
$$v_L = L\frac{\Delta i_L}{\Delta t} \quad \rightarrow \quad \Delta i_L = \frac{v_L \Delta t}{L} = \frac{V_d D}{f_{sw} L} = \frac{9V}{63\mu H} \cdot \frac{0.727}{100kHz} = 1.04A$$

The average input current is calculated as:

$$P_o = P_d \rightarrow V_o I_o = V_d I_d \rightarrow I_o = \frac{V_d I_d}{V_o} = \frac{I_d}{V_o}\frac{V_o(1-D)}{D} \quad \rightarrow \quad \frac{I_o}{I_d} = \frac{(1-D)}{D} \quad \rightarrow \quad I_d = \frac{D I_o}{(1-D)} = \frac{0.727 \cdot 2.5A}{(1-0.727)} = 6.66A$$

$$I_L = I_o + \frac{I_o D}{1-D} = 2.5A + \frac{2.5A \cdot 0.727}{1-0.727} = 9.16A$$

CCM is valid since $I_L > (\Delta i_L/2 = 0.52A)$.



Since the input filter is well dimensioned, the current drawn from the DC-source can be considered a purely DC-current. If a pure DC-current is flowing from the DC-source, the entire ripple current must flow through the capacitor. The rms-value of the capacitor current can be calculated according to the definition

$$I_{C(RMS)} = \sqrt{\frac{1}{10\mu}\int_0^{10\mu} i_c^2(t)dt} =$$

The definition for Simpsons rule is:

$$\frac{1}{T}\int_{t_0}^{t_0+T} f(x)dx = \frac{1}{6}\left(f(t_0) + 4 \cdot f\left(t_0 + \frac{T}{2}\right) + f(t_0+T)\right)$$

The RMS-current can now be calculated as:

$$I_{C(RMS)} = \sqrt{\frac{1}{10\mu}\left(7.27\mu\frac{1}{7.27\mu}\int_0^{7.27\mu} i_c^2(t)dt + 2.73\mu\frac{1}{2.73\mu}\int_{7.27\mu}^{10\mu} i_c^2(t)dt\right)} =$$

$$= \sqrt{\frac{7.27\mu}{10\mu}\frac{1}{6}\left(\left(9.16A - \frac{1.04A}{2} - 6.66A\right)^2 + 4(9.16A - 6.66A)^2 + \left(9.16A + \frac{1.04A}{2} - 6.66A\right)^2\right) + \frac{2.73\mu}{10\mu}(6.66A)^2}$$

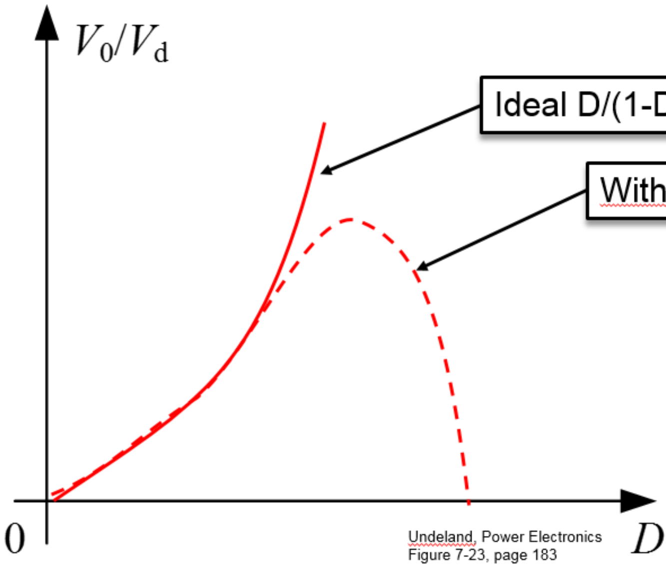
$$= 4.08A$$

The power dissipation in the capacitor can now be calculated as:

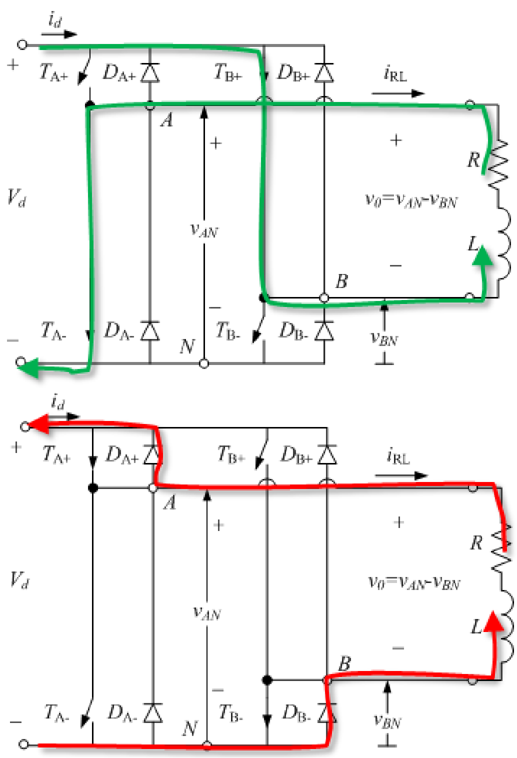
$$P_C = R_{ESR} I_{C(RMS)}^2 = 120m\Omega \cdot 4.08A^2 = 2.00W$$

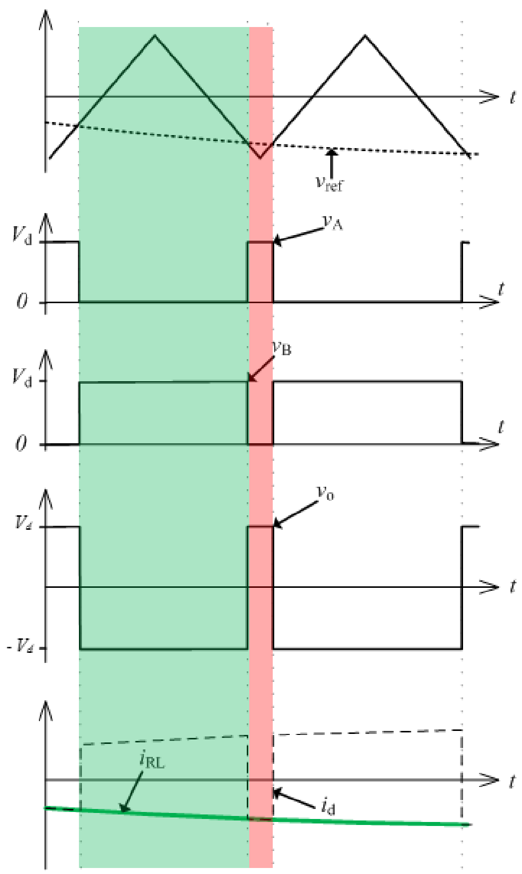
3) For the boost converter in (1), draw a graph of V_o/V_d as a function of the duty-cycle (D) with and without ideal components. Also, explain why the two graphs differ. (3p)

When ideal components are condsidered, a very high output voltage would be obtained if a high duty-cycle is used. This is not possible if non-ideal components are used. The main limitation is that when the swtich is closed, we would store a very large amount of energy in the inductor. This energy shall then be transferrred to the load during a ver short time when the switch is not conducting. This is not possible due to limitations (resistances) in the circuit.



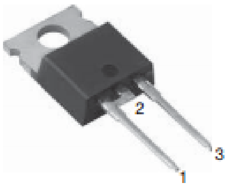
4) The single phase inverter below is operating with PWM bipolar switching. For the specified time instant (v_{ref} and i_{RL} are given), draw the resulting voltage over the load (v_0) and the current drawn from the source (i_d). Also mark the current paths during each time interval. (4p)



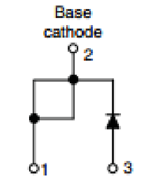


5) **The single phase inverter in (4) is operated in square wave mode ($V_d = 200V$ and 50Hz). Calculate the resulting temperature rise in diode D_{B-} if the load is purely inductive. The output current is assumed to have a peak value of 50A and the diode is mounted to a heat-sink with $R_{thSA} = 4.9°C/W$. The datasheet for the diode is attached below. (5p)**

Fast Soft Recovery Rectifier Diode, 20 A



TO-220AC



FEATURES

- 150 °C max operating junction temperature
- Low forward voltage drop and short reverse recovery time
- Designed and qualified according to JEDEC-JESD47
- Material categorization: For definitions of compliance please see www.vishay.com/doc?99912

RoHS COMPLIANT
HALOGEN FREE

PRODUCT SUMMARY	
Package	TO-220AC
$I_{F(AV)}$	20 A
V_R	800 V, 1000 V, 1200 V
V_F at I_F	1.31 V
I_{FSM}	320 A
t_{rr}	95 ns
T_J max.	150 °C
Diode variation	Single die
Snap factor	0.6

APPLICATIONS

These devices are intended for use in output rectification and freewheeling in inverters, choppers and converters as well as in input rectification where severe restrictions on conducted EMI should be met.

DESCRIPTION

The VS-20ETF... fast soft recovery rectifier series has been optimized for combined short reverse recovery time and low forward voltage drop.
The glass passivation ensures stable reliable operation in the most severe temperature and power cycling conditions.

THERMAL - MECHANICAL SPECIFICATIONS					
PARAMETER		**SYMBOL**	**TEST CONDITIONS**	**VALUES**	**UNITS**
Maximum junction and storage temperature range		T_J, T_{Stg}		- 40 to 150	°C
Maximum thermal resistance, junction to case		R_{thJC}	DC operation	0.9	°C/W
Maximum thermal resistance, junction to ambient		R_{thJA}		62	
Typical thermal resistance, case to heatsink		R_{thCS}	Mounting surface, smooth and greased	0.5	
Approximate weight				2	g
				0.07	oz.
Mounting torque	minimum			6 (5)	kgf · cm (lbf · in)
	maximum			12 (10)	
Marking device			Case style TO-220AC	20ETF08 20ETF10 20ETF12	

Due to the purely inductive load, the output current will be triangular shaped. The diode will conduct for 25% of the total time and during this time, the current will start at 0A and increase linearly to 50A.

$$I_{diode(AVG)} = \frac{1}{20m}\int_0^{20m} i_{diode}(t)dt =$$

The definition for Simpsons rule is:

$$\frac{1}{T}\int_{t_0}^{t_0+T} f(x)dx = \frac{1}{6}\left(f(t_0) + 4 \cdot f\left(t_0 + \frac{T}{2}\right) + f(t_0 + T)\right)$$

The AVG-current can now be calculated as:

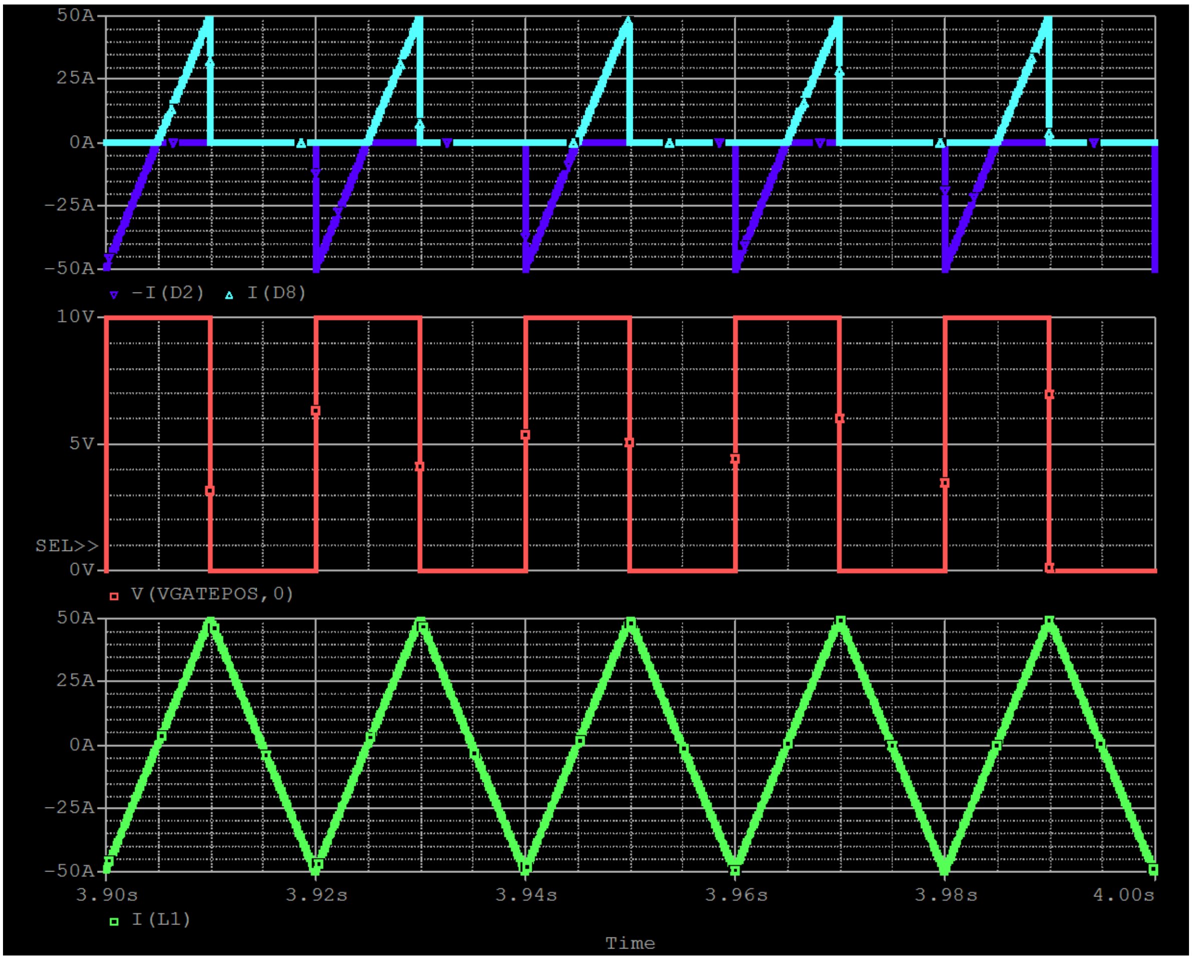
$$I_{C(RMS)} = \frac{1}{20m}\left(\frac{5m}{5m}\int_0^{5m} i_c(t)dt\right) =$$

$$= \frac{5m}{20m}\frac{1}{6}\left(0A + 4\left(\frac{50A}{2}\right) + 50A\right) = 6.25A$$

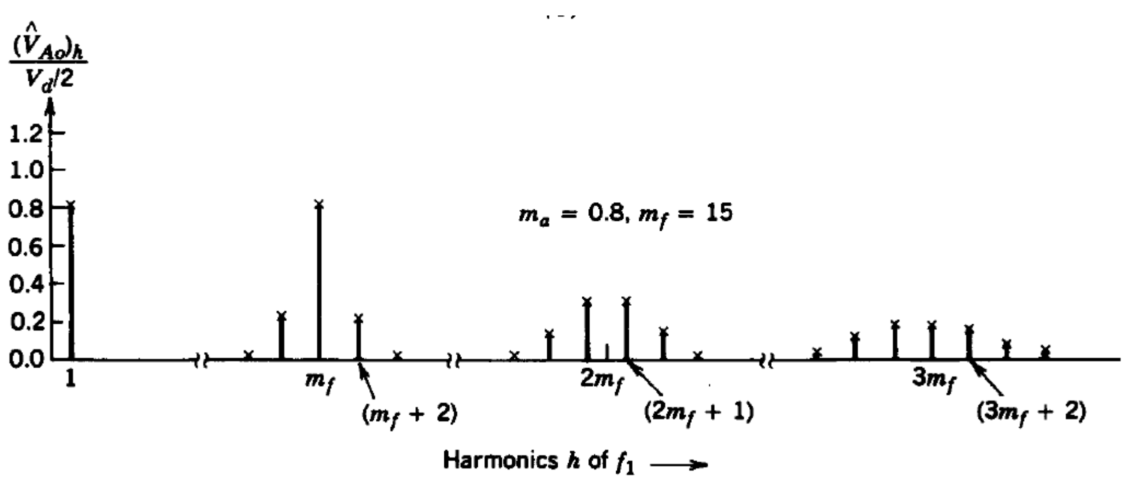
The power dissipation in the capacitor can now be calculated as:

$$P_{diode} = V_F I_{diode(AVG)} = 1.31V \cdot 6.25A = 8.2W$$

$$\Delta T_{diode} = P_{diode} \cdot R_{thCA} = 8.2W \cdot (0.9 + 0.5 + 4.9)°C/W = 51.7°C$$

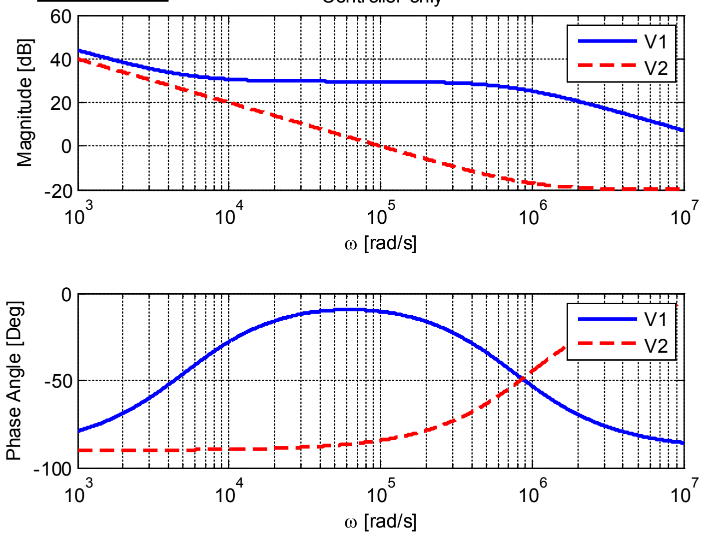


6) **For the single phase inverter in (4), sketch the resulting harmonic spectrum for $m_a = 0.8$ and $m_f = 15$. Mark the amplitudes for the specified frequencies (and sidebands) in the diagram. (4p)**

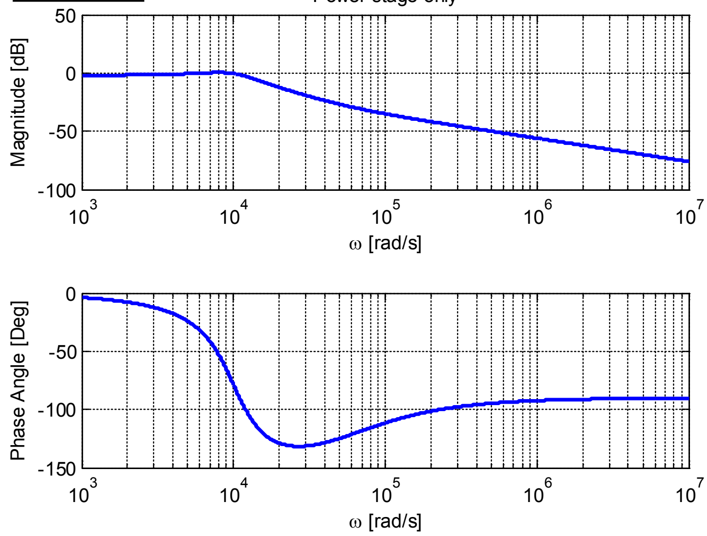


7) **The bode-plots below represent the transfer function of the power stage of a buck converter (plot (*a*)) and the transfer functions of two different controller structures (plot (*b*)). Which controller (V1 or V2) is best to use for the converter in plot (*b*)? Sketch the resulting transfer functions of the controller and the power stage and explain which controller shall be used. Empty bode plots can be found at the end of the exam. (4p)**

Plot (*a*)



Plot (*b*)

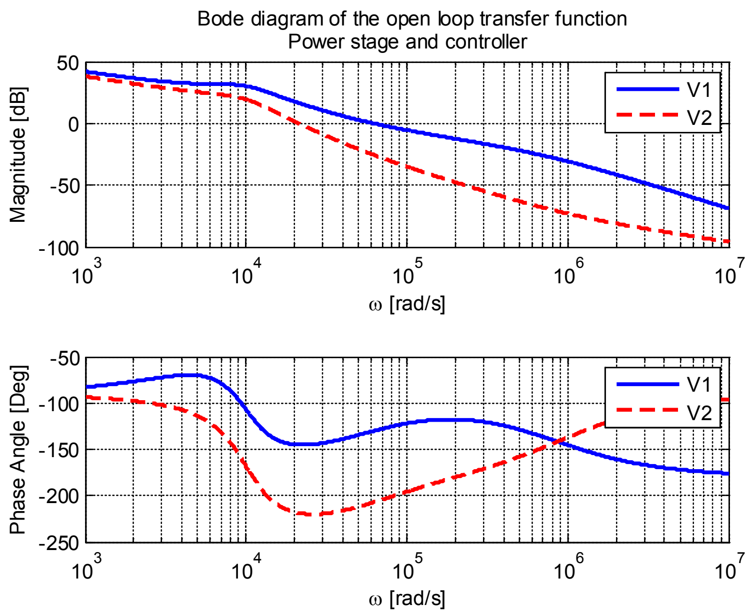


For controller V2, the gain goes toward infinity with a slope of -20dB/decade for low frequencies. After the crossover frequency (in this example approximately 10^6 rad/s), the gain is negative and flat. If the open loop transfer function of the entire system (controller circuit plus converter) is plotted, instabilities can be detected since the phase will drop to more than -180deg when the gain is still positive.

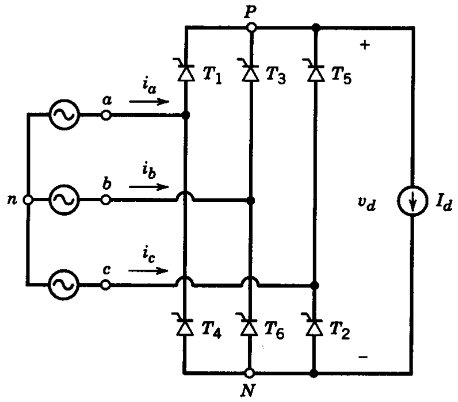
One of the reasons for the instability is that we in this case we have a more complex system; a second order system with an extra zero. A more suitable controller is proposed by Undeland:

$$T_c(s) = \frac{A}{s}\frac{s+\omega_z}{s+\omega_p}$$

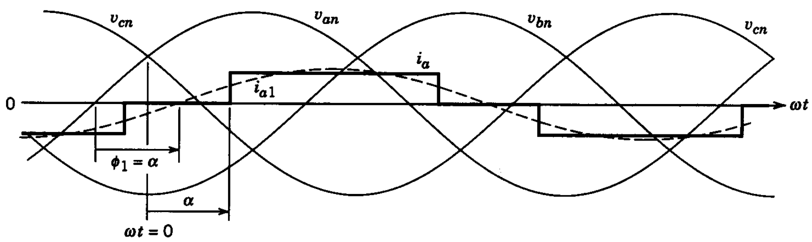
The main idea with this type of controller is to give an increased phase boost around the crossover-frequency. The bode plot for both types of regulators are plotted below. Note the increase in phase from -90° to ~0° around 10^5 rad/s. This phase boost is needed in order to lift the phase from the power stage so that it does not exceed the desired phase margin.



8) A three-phase thyristor rectifier (inverter) is loaded with a constant DC-side current ($I_d = 75A$). The phase voltages are $260V_{(RMS)}$ and the resulting average DC-side voltage is $V_d = 370V$. Draw the phase voltage (v_a), the resulting line current (i_a) and calculate its RMS-value. (3p)



The line current with its fundamental frequency component can be drawn as follows.



The RMS-value of the line current can be calculated as:

$$I_{RMS} = \sqrt{\frac{240}{360}} I_d = \sqrt{\frac{240}{360}} 75A = 61A$$

9) For the thyristor rectifier in (7), derive an expression for the resulting DC-link voltage as a function of known quantities. Also, calculate the needed delay angle (α) to obtain the specified DC-link voltage (5p)

The average DC-link voltage can be calculated as:

$$V_d = \frac{1}{\pi/3} \int_{-\pi/6}^{\pi/6} \sqrt{2} \cdot V_{LL} \cdot \cos(\omega t) d\omega t =$$

$$= \frac{3 \cdot \sqrt{2} \cdot V_{LL}}{\pi} \left[\sin(\omega t)\right]_{-\pi/6}^{\pi/6} = \frac{3 \cdot \sqrt{2} \cdot V_{LL}}{\pi} \left(\frac{1}{2} - \frac{-1}{2}\right) \rightarrow$$

$$V_{d(AVG)} = \frac{3 \cdot \sqrt{2} \cdot V_{LL}}{\pi} = 1.35 \cdot V_{LL}$$

The reduction in output voltage (A_α) due to the delay angle can be calculated as:

$$A_\alpha = \int_0^\alpha \sqrt{2} V_{LL} \sin(\theta) d\theta = \sqrt{2} V_{LL} [-\cos(\theta)]_0^\alpha = \sqrt{2} V_{LL} (1 - \cos(\alpha))$$

Now can the dc-link voltage be expressed as:

$$V_d = V_{d0} - \frac{3}{\pi}(A_\alpha) = \frac{3\sqrt{2}V_{LL}}{\pi} - \frac{3}{\pi}\left(\sqrt{2}V_{LL}(1-\cos(\alpha))\right) = \frac{3\sqrt{2}V_{LL}}{\pi}\cos(\alpha)$$

Under the assumption that the converter is lossless, the following relation holds:

$$P_{DC} = P_{DC} \quad \rightarrow \quad V_d I_d = 1.35 V_{LL} I_d \cos\alpha$$

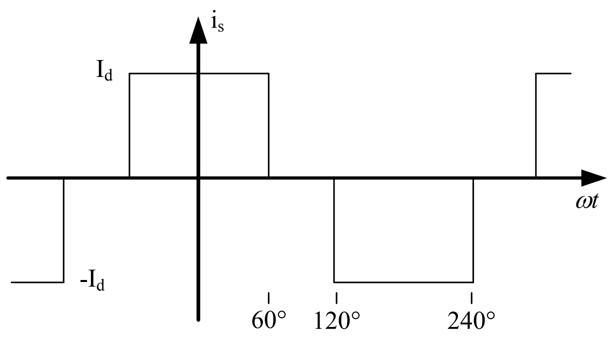
Therefore, the delay angle α can be calculated as

$$\alpha = \arccos\frac{V_d}{1.35V_{LL}} = \arccos\frac{370V}{1.35 \cdot 260V \cdot \sqrt{3}} = 52.5°$$

10) If the thyristor rectifier in (7) is operated with zero delay angle ($\alpha = 0°$), calculate the fundamental frequency component in the line current and the resulting power factor (PF). Why does the rectifier consume reactive power? (4p)

If the thyristor rectifier is operated with zero degrees delay angle, the resulting DC-link voltage will be:

$$V_d = 1.35V_{LL} = 1.35 \cdot 260V \cdot \sqrt{3} = 608V$$



The current will be a quasi-square wave and by centering the waveform, it becomes an even quarter-wave function:

$$a_n = \frac{4}{\pi}\int_0^{\pi/2} f(t)\cos(n\omega t)\,d\omega t = \frac{4}{\pi}\int_0^{\pi/3} I_d\cos(n\omega t)\,d\omega t = \frac{4}{\pi}\frac{I_d}{n}[\sin(n\omega t)]_0^{\pi/3} = \frac{4}{\pi}\frac{I_d}{n}\left(\underbrace{\frac{\sqrt{3}}{2}}_{\pm} - 0\right) = \underbrace{\frac{2\sqrt{3}}{\pi}}_{\pm}\frac{I_d}{n}$$

(for n=odd and $a_n = 0$ for n=even)

$b_n = 0$ for all n

The RMS-value of the fundamental component can be calculated as:

$$I_{s(1)} = \frac{a_1}{\sqrt{2}} = \frac{\sqrt{6}}{\pi}I_d = \frac{\sqrt{6}}{\pi}75A = 58.5A$$

The line current will contain a lot of harmonics, and since only voltage and currents with the same frequency (assuming that the line voltage is ideally sinusoidal) will contribute to the active power transfer, the PF will be reduced. The even and triplen harmonics are zero due to symmetry and the three phase system. Since $i_{s(1)}$ is in phase with the phase voltage

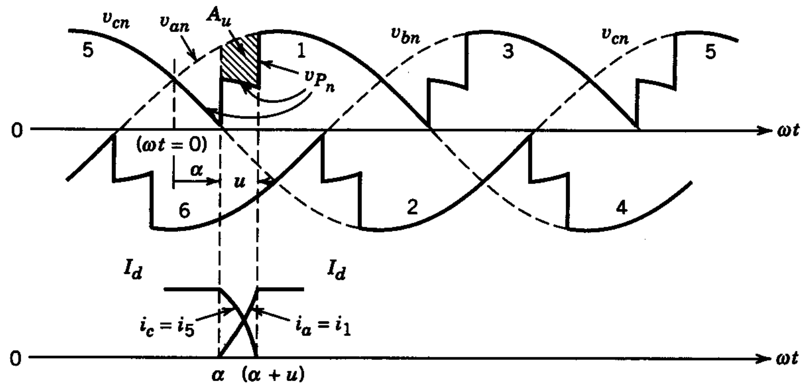
$$DPF = 1.0$$

And therefore

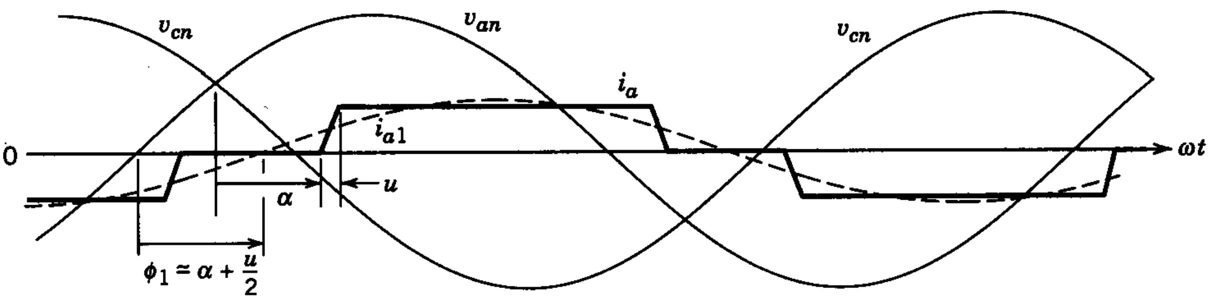
$$PF = \frac{3}{\pi} = 0.955$$

11) If the source inductance is taken into account in the rectifier in (7), draw the line currents in two phases during the commutation, e.g. phase c (i_5) and phase a (i_1). How will the commutation affect the PF and/or the DPF of the rectifier? (3p)

Due to the source inductance, the current can't shift from one thyristor to another thyristor instantaneously. The commutation process will take some time that depends on the current, voltage and the inductance. During the commutation, the current will increase exponentially in one phase and decrease exponentially in the other phase.



Due to the commutation, the phase shift of the fundamental frquency component in the current will increase. The DPF will decrease due to the increased phase angle but the inductance also reduces tha magnitude of the harmonic currents (see section 6-4-2-1 in Undeland).



12) In a flyback converter, the transformer has a primary magnetizing inductance of $100\mu H$. Calculate the air-gap length needed to decrease the inductance to $50\mu H$. Explain also why an air-gap is needed in the transformer. (4p)

Mean path length (l_m): 2π cm
Cross sectional area (A_m): $1cm^2$
Turns ratio: N_1: N_2: N_3 $= 40:40:40$ turns.

From the formula sheet:

$$R = \frac{l}{A\mu_r\mu_0} \qquad NI = R\phi \qquad N\phi = LI$$

Thereby: $L = \frac{N^2}{R}$ and the relative permeability of the core is:

$$\mu_r = \frac{L \cdot l_m}{N^2 A_m \mu_0} = \frac{100u \cdot 2\pi \cdot 10^{-2}}{40^2 \cdot 1 \cdot 10^{-4} \cdot 4\pi \cdot 10^{-7}} = 31.25$$

The new reluctance must be doubled compared with the original case, since L should be halved.

$$R_{tot-new} = 2R_{old} = 2\frac{N^2}{L} = R_{core} + R_{airgap} = \frac{l_m - l_g}{A_m \mu_r \mu_0} + \frac{l_g}{A_m \mu_0}$$

which gives

$$l_g = \frac{2N^2 A_m \mu_0 \mu_r - L l_m}{(\mu_r - 1)L} = \frac{2N^2 A_m \mu_0 \mu_r - L l_m}{(\mu_r - 1)L} = \frac{2 \cdot 40^2 \cdot 1 \cdot 10^{-4} \cdot 4\pi \cdot 10^{-7} \cdot 31.25 - 100u \cdot 2\pi \cdot 10^{-2}}{(31.25 - 1)100u} = 2.1mm$$

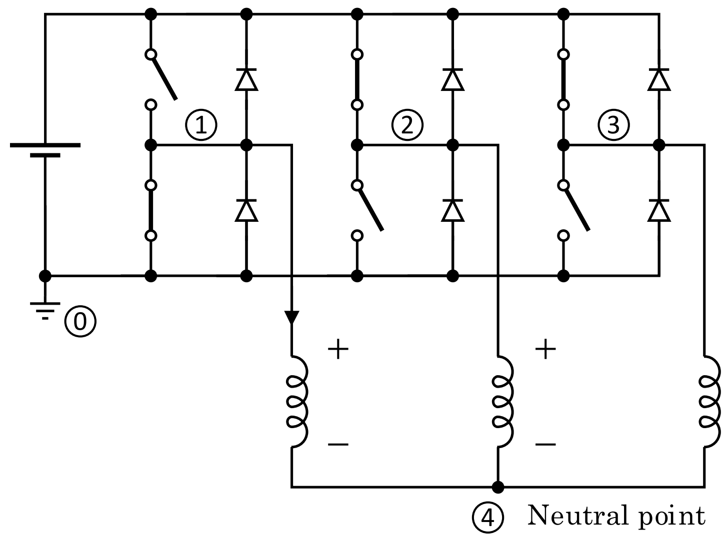
Where L represents the inductance without air-gap.

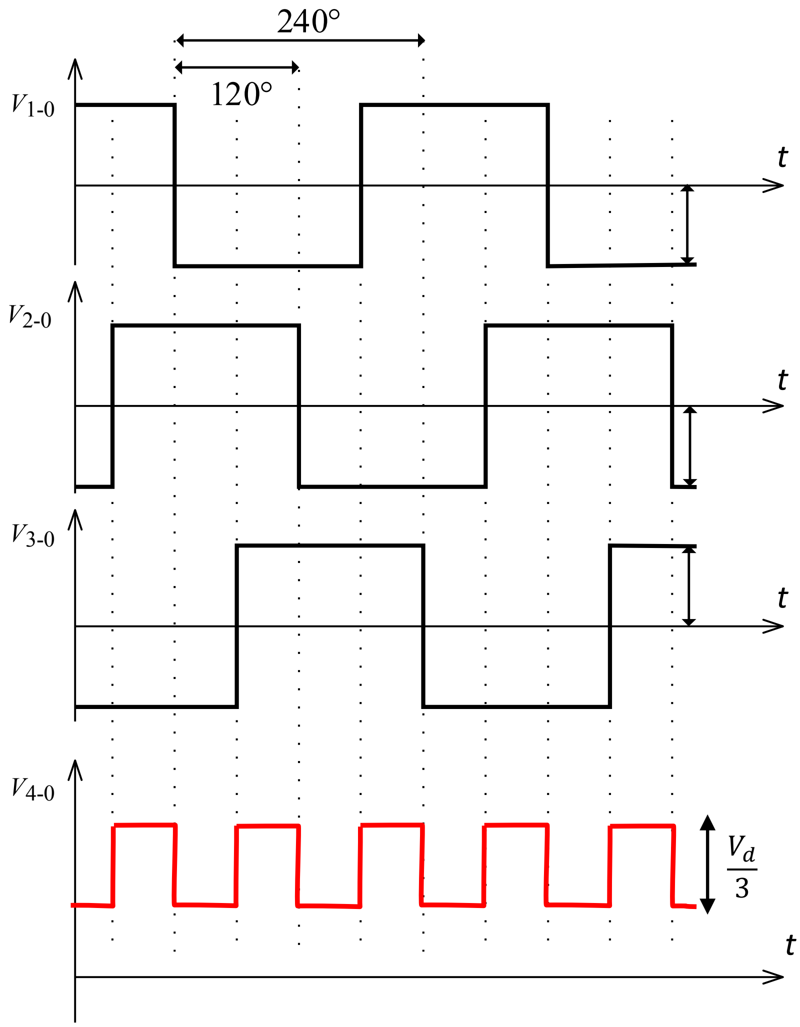
An air-gap reduces the magnetic flux for a given current. Hence, saturation of the core material can be avoided for higher currents. Even if the inductance becomes lower, more energy can be stored in the flyback transformer before the non-linear region of the core material is reached:

$$W = \frac{1}{2} L I^2$$

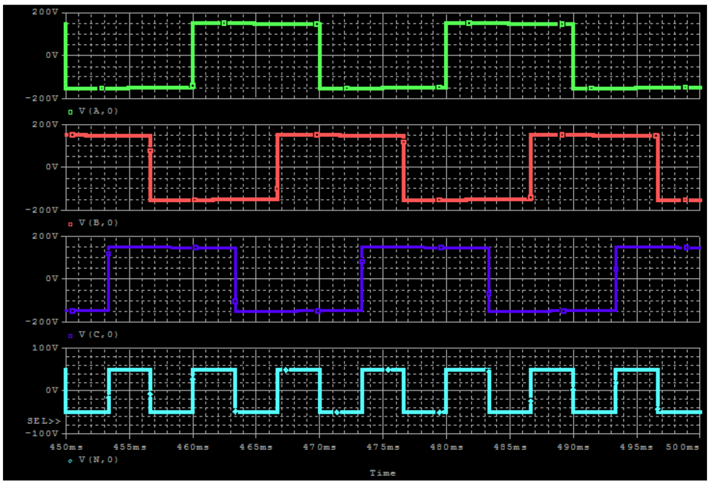
13) The three phase inverter below is operating with square wave mode. Draw the resulting voltages in the midpoint of each phase leg (v_{1-0}, v_{2-0} and v_{3-0}) and the neutral point voltage (v_{4-0}). (4p)

The difference from the derivation in lecture 12 is that the negative DC-link is grounded instead of the mid-point of the DC-link. This makes the neutral point voltage to have an offset of half the DC-link voltage.

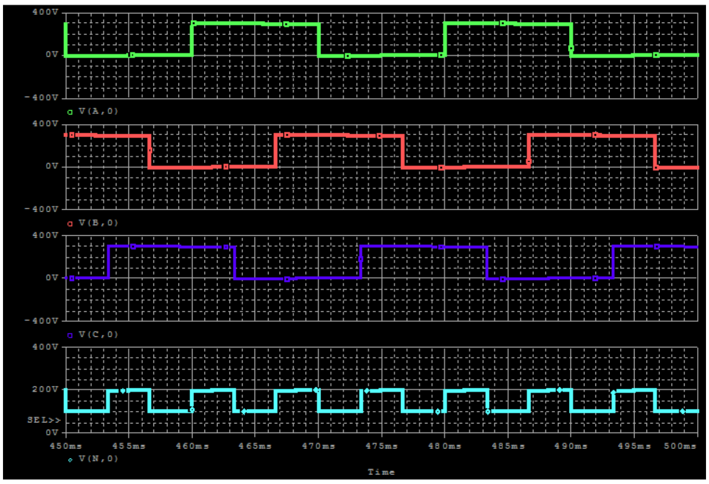




GND and midpoint of the DC-link



GND at negative rail of the DC-link



Formulas for Examination in Power Electronic Converters (ENM060)

Table 3-1 Use of Symmetry in Fourier Analysis

Symmetry	*Condition Required*	a_h and b_h
Even	$f(-t) = f(t)$	$b_h = 0 \quad a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\,d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0 \quad b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\,d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even h $a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\,d(\omega t)$ for odd h $b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\,d(\omega t)$ for odd h
Even quarter-wave	Even and half-wave	$b_h = 0$ for all h $a_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\cos(h\omega t)\,d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all h $b_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\sin(h\omega t)\,d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$

Definition of RMS-value:

$$F_{RMS} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} f(t)^2 dt}$$

Definition of RMS-value with Fourier-series:

$$F_{RMS} = \sqrt{F_0^2 + \sum_{n=1}^{\infty} F_n^2} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}\right)^2}$$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \qquad \sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \qquad \cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right) \qquad \sin(\alpha)\cos(\beta) = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right)$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax), \quad \int x\sin(ax)dx = \frac{1}{a^2}\left(\sin(ax) - ax\cos(ax)\right), \quad \int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int x\cos(ax)dx = \frac{1}{a^2}\left(\cos(ax) + ax\sin(ax)\right)$$

$$PF = \frac{P}{S} = \frac{V_s I_{s1}\cos\phi_1}{V_s I_s}, \quad DPF = \cos\phi_1, \quad \%THD_i = 100\frac{I_{dis}}{I_{s1}} = 100\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = 100\sqrt{\sum_{h\neq 1}\left(\frac{I_{sh}}{I_{s1}}\right)^2}$$

Electromagnetics

$$e = \frac{d}{dt}\psi \qquad \psi = N\phi \qquad \phi = BA \qquad R = \frac{l}{A\mu_r\mu_0} \qquad L = \frac{\Psi}{i}$$

$$NI = R\phi = mmf \qquad N\phi = LI \qquad L = A_L N^2 \qquad W = \frac{1}{2}LI^2$$

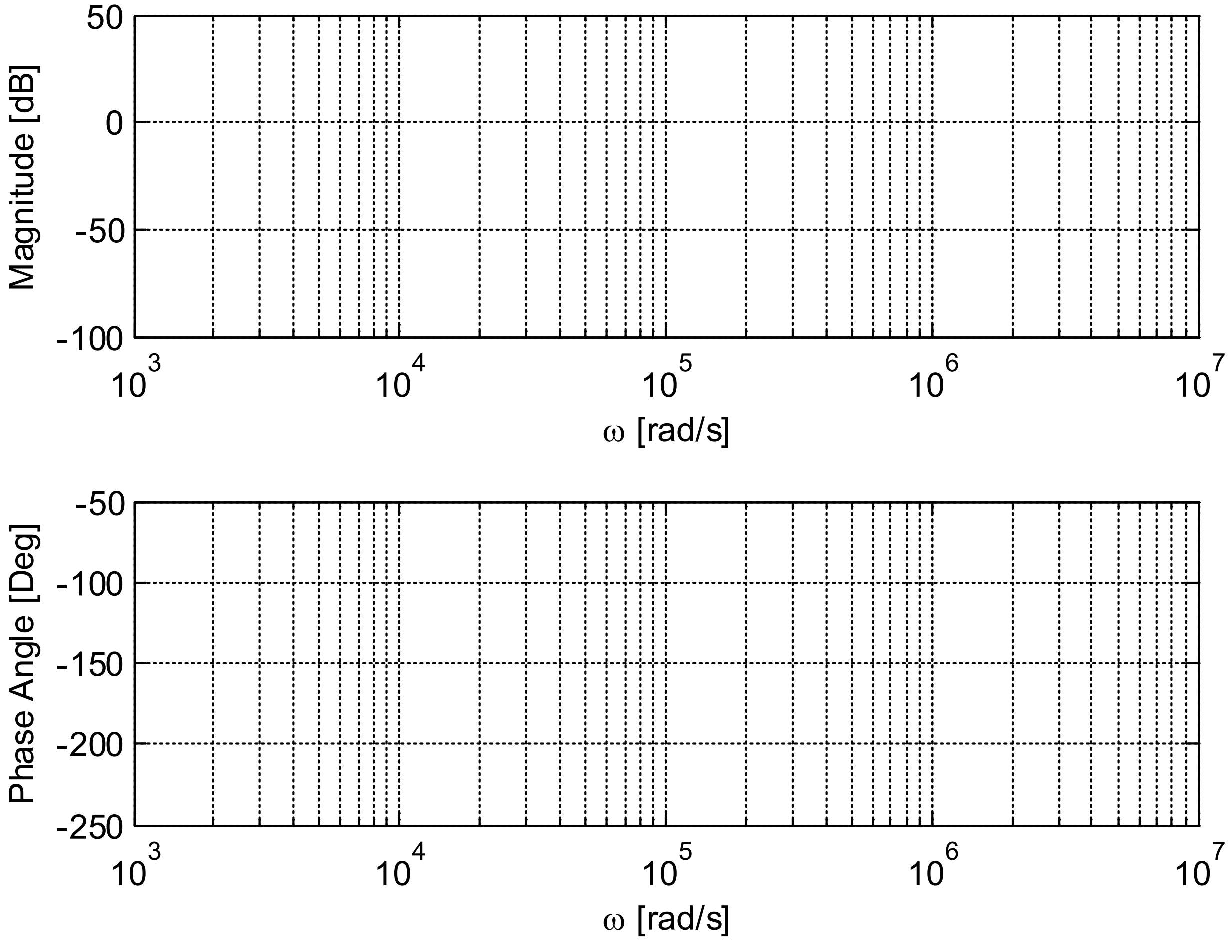
Simpson's rule

Let $f(x)$ be a polynomial of maximum third degree, this means

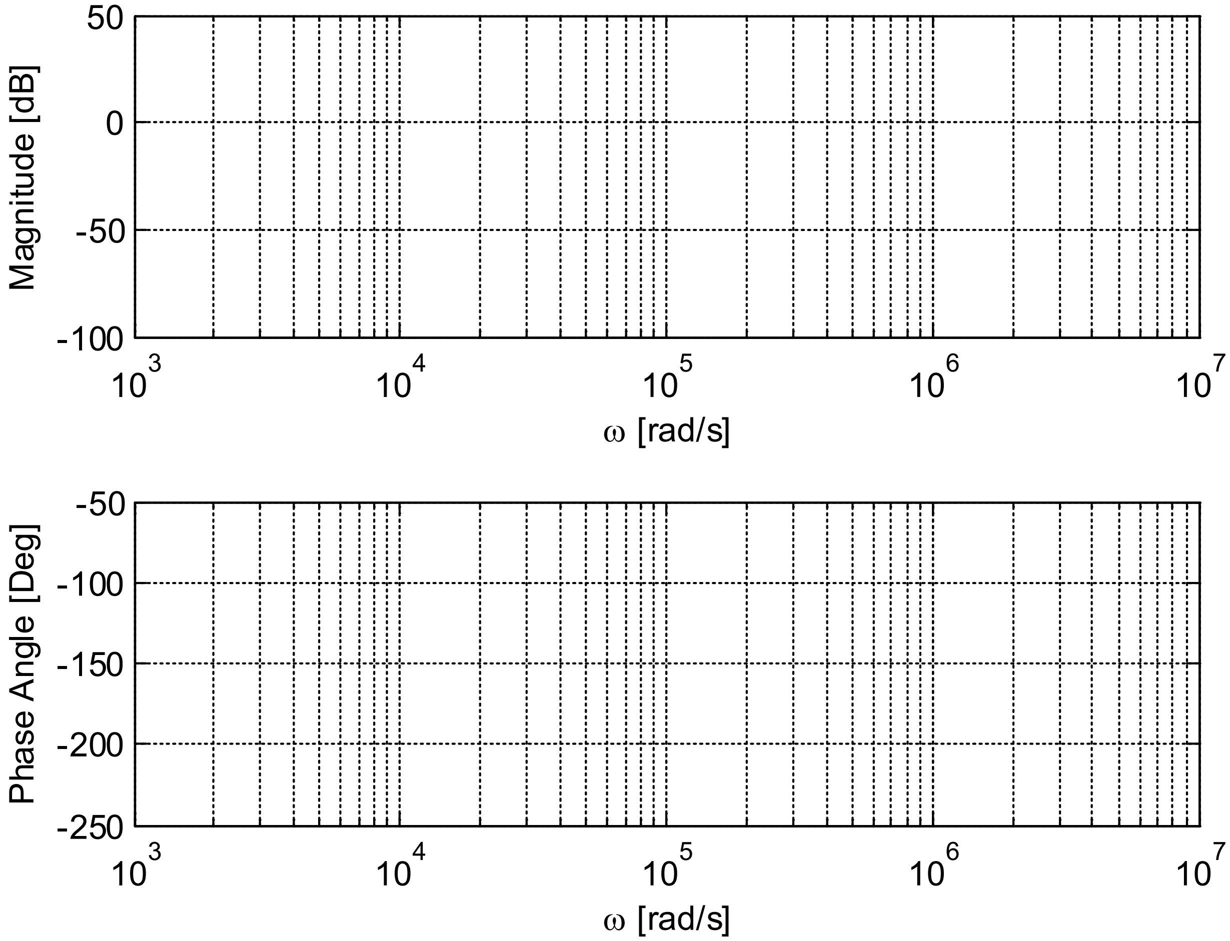
$$f(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

For this function the integral can be calculated as

$$\frac{1}{T}\int_{t_0}^{t_0+T} f(x)dx = \frac{1}{6}\left(f(t_0) + 4f(t_0 + \frac{T}{2}) + f(t_0 + T)\right)$$



Magnitude [dB]
50
0
-50
-100
10^3
10^4
10^5
10^6
10^7
ω [rad/s]
Phase Angle [Deg]
-50
-100
-150
-200
-250
10^3
10^4
10^5
10^6
10^7
ω [rad/s]


50
0
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Magnitude [dB]
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10^4
10^5
10^6
10^7
ω [rad/s]