SVAR TIL Tentamen i

Elektriska kretsar och Signaler del B, EMI190 (D2)

Signaler & System, ESS050 (Z2)

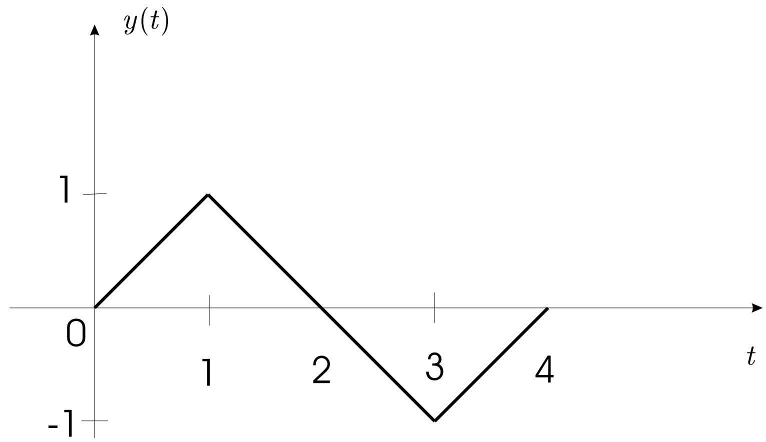
17 augusti 2000

1. a) Laplace transform the equation, and partial fraction expansion, leads to

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s} + \frac{4}{s+2}$$

so that $h(t) = (2 + 4e^{-2t})u(t)$.

b) Applying the sifting property ($\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)dt = x(t)$) of the dirac delta function, and the fact that the system is LTI, signal out is given in the figure below.



2. Take the Laplace transform (table!) of signals $x_1(t)$ and $y_1(t)$. Computing the ratio of $Y_1(s)$ and $X_1(s)$ gives the unknown transfer function, i.e. $X_1(s) = \frac{1}{s+1}$, and $Y_1(s) = \frac{-4}{(s+2)^2} - \frac{1}{s+2} = \frac{-(s+6)}{(s+2)^2}$. Taking the ratio leads to

$$H(s) = \frac{Y_1(s)}{X_1(s)} = \frac{-(s+6)(s+1)}{(s+2)^2}.$$

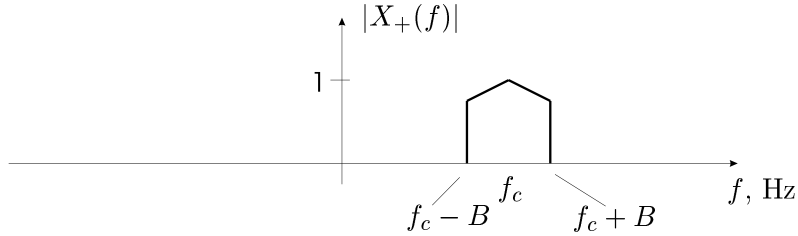
Now, we can easily find $Y_2(s) = H(s)X_2(s) = \frac{-(s+1)}{(s+2)^2}$, where $X_2(s) = \frac{3}{s+6}$ (from table). Finally, take the inverse Laplace transform of $Y_2(s)$, together with a partial fraction expansion, to get the answer $y_2(t) = 3te^{-2t}u(t) - 3e^{-2t}u(t)$.

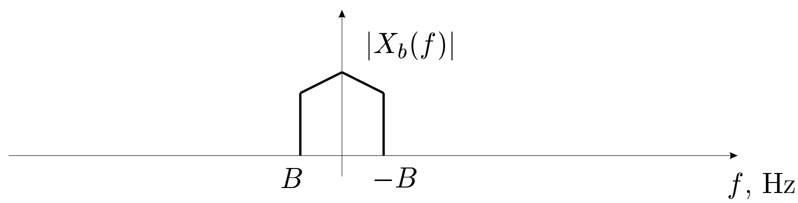
3. a)

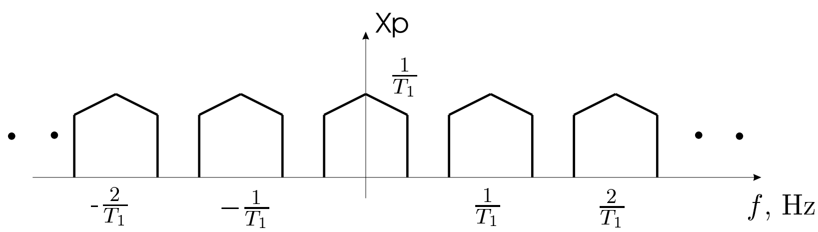
b)

4. a) Sampling-frequency $f_s = 2 \cdot 37.8$ kHz, according to the sampling theorem. Use for instance $f_s = 100$ kHz.

b) The number of samples must be sufficiently large so that the frequency resolution is better than $(37.8 - 37.6)/2 = 0.1$ kHz. We include the factor of 2 so that we get at least one sample between the two expected peaks (since the purpose is to separate the two peaks).Using N samples, the frequency resolution is 100 kHz/N. So use $N = 100/0.1 = 1000$ samples. (Note that performing this in e.g. Matlab, the samples will be zero-padded to $2^10 = 1024$, in order for the FFT algorithm to be computationally efficient.)



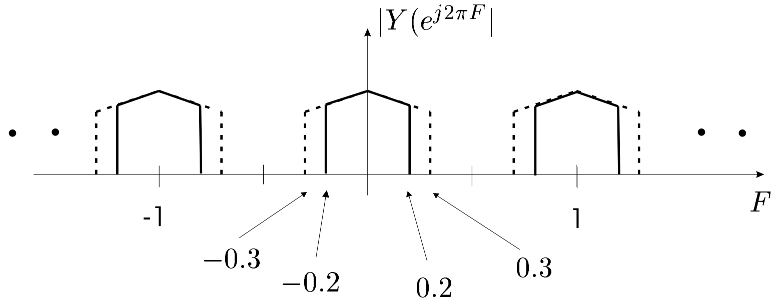




5. a)

Since the spectrum of the signal $x_+(t)$ is non-symmetric around zero, we conclude that $x_+(t)$ must be a complex signal (the operation carried out by the filter $\Phi(f)$ is referred to as a Hilbert transform).

b) The multipliction by the pulse train results in repeated spectra every $n\frac{1}{T_1} = 10$ kHz (corresponding to $F = 1$ in relative frequency). With a bandwidth of $B = 3000$ Hz, we get a relative bandwidth of $B_F = 3000/10000 = 0.3$ in relative frequency. Since the bandwidth of the filter is less than this, part of the signal will be destroyed passing through the filter; It will consequenty be impossible to reconstruct the original signal $x_c(t)$ from $y[n]$. The spectrum of $y[n]$ is shown in the figure, with the spectrum of $x[n]$ shown as dotted lines.



c) Any sampling process results in repeated spectra every $1/T_s$, where T_s is the sampling interval. Thus, if we use $T_2 = 0.10$ ms, the result will be identical (apart from a simple amplitude scaling) to that obtained above.