$R_1 = 2{,}0\ \Omega$

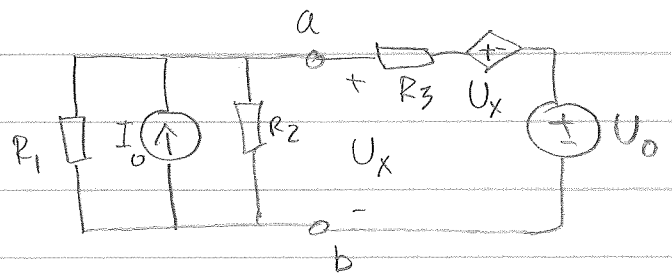
$R_2 = 2{,}0\ \Omega$

$R_3 = 4{,}0\ \Omega$

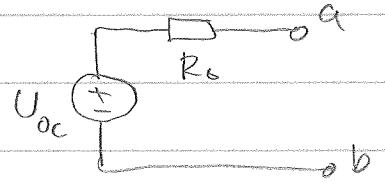
$U_0 = 18\ V$

$I_0 = 3{,}0\ A$

1/

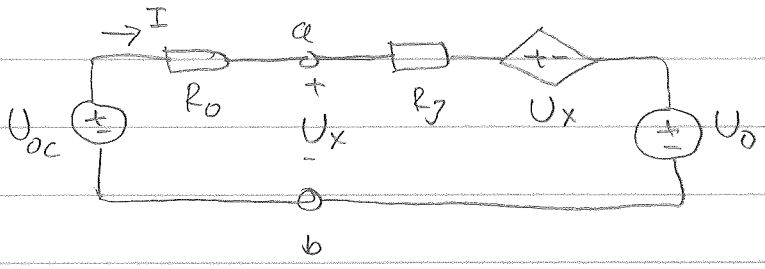


Tvåpolsomvandla



$$R_0 = R_1 // R_2 = \frac{2 \cdot 2}{2+2} = 1\ \Omega$$

$$U_{OC} = I_0 \cdot R_0 = 3 \cdot 1 = 3\ V$$



$$\text{KVL:} \begin{cases} -U_{OC} + IR_0 + IR_3 + U_X + U_0 = 0 & (1) \\ -U_{OC} + IR_0 + U_X = 0 & (2) \end{cases}$$

(2): $I = \dfrac{U_{OC} - U_X}{R_0}$

(1): $-U_X + IR_3 + U_X + U_0 = 0 \Rightarrow I = -\dfrac{U_0}{R_3}$

$$-\frac{U_0}{R_3} = \frac{U_{OC}}{R_0} - \frac{U_X}{R_0}$$

$$U_X = R_0\left(\frac{U_{OC}}{R_0} + \frac{U_0}{R_3}\right) = U_{OC} + \frac{R_0}{R_3}U_0 =$$

$$= 3 + \frac{1}{4} \cdot 18 = 3 + \frac{9}{2} = \frac{15}{2} = 7{,}5\ V$$

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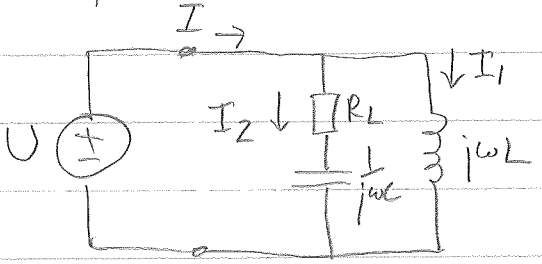
2/

jω-transf. kretsen



$I_1 = \hat{I}_1 \angle \Phi_1$

$I_2 = \hat{I}_2 \angle \Phi_2$

$U = \hat{U} \angle 0°$

$\omega = 10^3$ rad/s

$\hat{I}_1 = 0{,}5$ A | $R_L = 1{,}0$ kΩ

$\hat{I}_2 = 1.0$ A | $C = 1.0$ μF

KCL; $I = I_1 + I_2$

$$U = I_1 \cdot j\omega L$$

$$I_1 = \frac{U}{j\omega L} = \left|\frac{U}{\omega L}\right| \angle -90° = \hat{I}_1 \angle -90° = -j \cdot \hat{I}_1$$

$$I_2 = \frac{U}{R_L + \frac{1}{j\omega C}} = \frac{U}{Z} = \frac{\hat{U}\angle 0°}{|Z| \angle \Phi_Z} = \hat{I} \angle -\Phi_Z$$

$$\Phi_Z = \arctan\left(\frac{-1}{\omega R_L C}\right) = -\arctan\left(\frac{1}{1000 \cdot 10^3 \cdot 10^{-6}}\right) = -45° = -\angle \Phi_Z$$

$$I = I_1 + I_2 = \hat{I}_1 \angle -90° + \hat{I}_2 \angle +45° =$$

$$= -j\hat{I}_1 + \hat{I}_2 \frac{1}{\sqrt{2}}(1+j) = \frac{\hat{I}_2}{\sqrt{2}} - j\left(\hat{I}_1 - \frac{\hat{I}_2}{\sqrt{2}}\right) =$$

$$= \frac{1}{\sqrt{2}} - j\left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} - j\left(\frac{1-\sqrt{2}}{2}\right) \approx 0{,}707 + j0{,}207 \approx$$

$$\approx 0{,}736 \angle 16{,}3°$$

Svar: $i(t) = 0{,}74 \cos(\omega t + 16{,}3°)$ A

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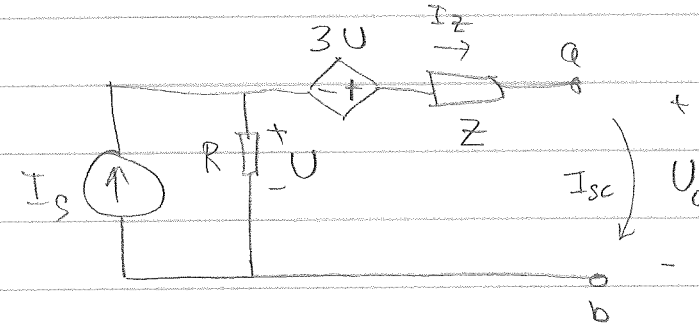
3/



$I_s = 2{,}0\angle 0°$

$R = 10\ \Omega$

$Z = j10\ \Omega$

o Kortslutn. ström ($I_z = I_{sc}$)

KVL: $-U - 3U + Z \cdot I_{sc} = 0$ (1)

KCL: $I_s = \frac{U}{R} + I_{sc}$ (2)

(2) $RI_s = U + RI_{sc} \Rightarrow U = R(I_s - I_{sc})$

(1) $I_{sc} = \frac{4U}{Z} = \frac{4R(I_s - I_{sc})}{Z}$

$Z I_{sc} + 4RI_{sc} = 4RI_s$

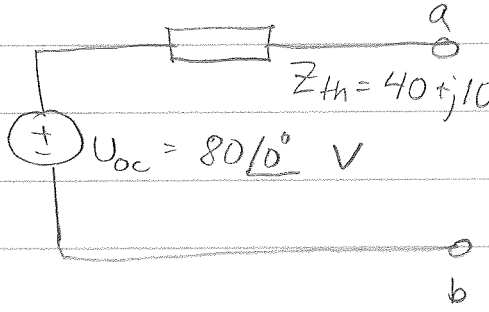
$$I_{sc} = \frac{4RI_s}{Z+4R} = \frac{I_s}{1+\frac{Z}{4R}} = \frac{2}{1+j\frac{10}{40}} = \frac{2}{1+j\frac{1}{4}}$$

o Tomgångsspänning ($I_z = 0$)

$$\begin{cases} U = RI_s \\ U_{oc} = 4U \end{cases} \Rightarrow U_{oc} = 4RI_s = 4 \cdot 10 \cdot 2 = 80\angle 0°$$

o Ekv. impedans $Z_{th} = \frac{U_{oc}}{I_{sc}} = \frac{80}{2}\left(1+j\frac{1}{4}\right) = 40 + j10$

Thevenins ekv. krets



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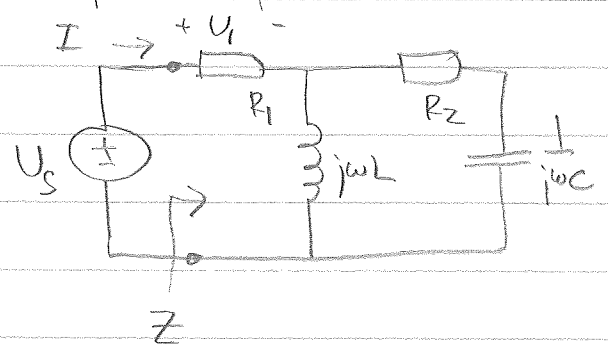
4/ jω-transf. kretsen



$R_1 = 1{,}0\ \Omega$

$R_2 = 2{,}0\ \Omega$

$L = 3{,}0\ H$

$C = 0{,}25\ F$

$u_s(t) = 16\cos(2{,}0t - 40^\circ)$

$\Rightarrow U_s = 16\angle -40^\circ$

$$Z = R_1 + j\omega L \,/\!/\, \left(R_2 + \frac{1}{j\omega C}\right) = R_1 + \frac{j\omega L\left(R_2 + \frac{1}{j\omega C}\right)}{j\omega L + R_2 + \frac{1}{j\omega C}} =$$

$$= R_1 + \frac{L/C + j\omega L R_2}{R_2 + j\left(\omega L - \frac{1}{\omega C}\right)} = 1{,}0 + \frac{3/0{,}25 + j\,2\cdot 3\cdot 2}{2 + j\left(2\cdot 3 - \frac{1}{2\cdot 0{,}25}\right)} =$$

$$= 1 + \frac{12 + j12}{2 + j4} = \ldots = 4{,}6 - j1{,}2$$

$$I = \frac{U_s}{Z} = \frac{16\angle -40^\circ}{4{,}6 - j1{,}2}$$

Effekt i R_1 : $S_1 = \frac{1}{2} U_1 I^* = \frac{1}{2} R_1 I I^* = \frac{1}{2} R_1 |I|^2 = P_1$

P_1 = Medeleffekt i R_1

$$P_1 = \frac{1}{2} R_1 |I|^2 = \frac{1}{2}\cdot 1\cdot \frac{16^2}{4{,}6^2 + 1{,}2^2} = 5{,}66\ W$$

Medeleffekt som upptas av L : $P_L = 0$ W

Svar: Medeleffekt i R_1 : $P_1 = 5{,}66$ W

" i L : $P_L = 0$ W

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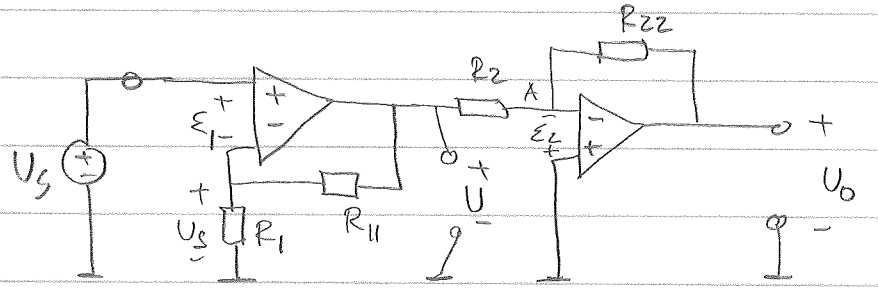
5/ Rita om kretsen något



Ideala op. först. Neg. återkopp. $\Big\}$ $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0$, $i_{op} = 0$

Inför "hjälpspänning" U.

Spänningsdelning: $U_s = U \dfrac{R_1}{R_1 + R_{11}}$

$$\text{KCL}_A: \quad \frac{U}{R_2} + \frac{U_0}{R_{22}} = 0 \quad \Rightarrow \quad U = -U_0 \frac{R_2}{R_{22}}$$

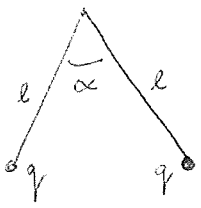
Eliminera U.

$$U_s = -U_0 \frac{R_2}{R_{22}} \cdot \frac{R_1}{R_1 + R_{11}}$$

$$\frac{U_0}{U_s} = -\frac{R_{22}}{R_2} \cdot \frac{R_1 + R_{11}}{R_1} = -\frac{R_{22}(R_1 + R_{11})}{R_1 R_2}$$

6) eem076 120521

Kraften mellan de laddade kulorna ska balanseras av tyngdkraften

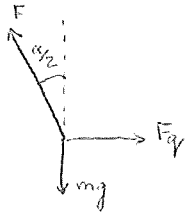


Kraften mellan laddningarna ges av Coulombs lag

$$F_q = \frac{q^2}{4\pi\varepsilon_0 r^2}$$

där r är avståndet mellan laddningarna

Kraftsituation



vid jämvikt: $mg\tan\frac{\alpha}{2} = \frac{q^2}{4\pi\varepsilon_0 (2\ell\sin\frac{\alpha}{2})^2}$

$$mg\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{q^2}{16\pi\varepsilon_0\ell^2\sin^2\frac{\alpha}{2}} \Rightarrow \frac{\sin^3\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{q^2}{16mg\pi\varepsilon_0\ell^2}$$

Antag $m = 1\,kg$
$\ell = 1\,m$
$q = 1\,\mu C$

och så utar vi små vinklar

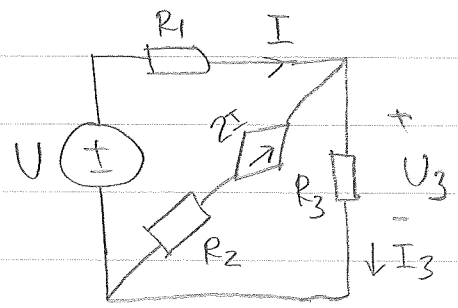
$$\sin^3\left(\frac{\alpha}{2}\right) \approx \left(\frac{\alpha}{2}\right)^3$$
$$\cos\frac{\alpha}{2} \approx 1$$

$$\left(\frac{\alpha}{2}\right)^3 = \frac{q^2}{16mg\pi\varepsilon_0\ell^2} \Rightarrow \alpha \approx 7^\circ$$

således är även små vinklar approximationen helt OK

Elektriska Kretsar och Fält
D1 eem076 120824

1.



$R_1 = R_3 = 3{,}0\ \Omega$

$R_2 = 5{,}0\ \Omega$

$U = 10\ V$

KCL: $I_3 = 2I + I = 3I$ (1)

KVL: $-U + IR_1 + 3I \cdot R_3 = 0$ (2)

Ω-lag: $U_3 = I_3 R_3 = 3IR_3$ (3)

(2): $I(R_1 + 3R_3) = U$

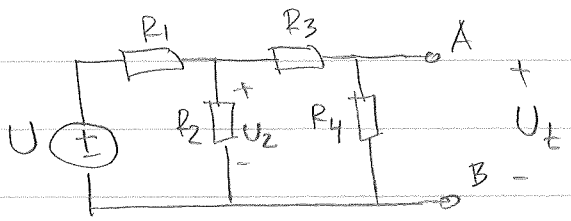
$$I = \frac{U}{R_1 + 3R_3}$$

(3): $$U_3 = \frac{3R_3 \cdot U}{R_1 + 3R_3} = \frac{U}{1 + \frac{R_1}{3R_3}}$$

$$U_3 = \frac{10}{1 + \frac{3}{3 \cdot 3}} = \frac{10}{1 + \frac{1}{3}} = \frac{10}{\frac{4}{3}}$$

$$U_3 = \frac{3 \cdot 10}{4} = \frac{30}{4} = \frac{15}{2}\ V$$

2.



$R_1 = 200\ \Omega$
$R_2 = 300\ \Omega$
$R_3 = 60\ \Omega$
$R_4 = 220\ \Omega$
$R_5 = 100\ \Omega$
$U = 120\ V$

a/ Tomgångsspänning U_t.

Sp. delning $U_2 = U \dfrac{R_2 /\!/ (R_3+R_4)}{R_1 + R_2 /\!/ (R_3+R_4)} =$

$= U \dfrac{\frac{R_2(R_3+R_4)}{R_2+R_3+R_4}}{R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}} = U \dfrac{1}{1 + \frac{R_1(R_2+R_3+R_4)}{R_2(R_3+R_4)}} =$

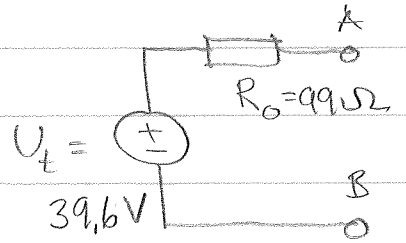
$= U \cdot \dfrac{1}{1 + \frac{200(300+60+220)}{300(60+220)}} = \dfrac{U}{1+\frac{29}{21}} = 0{,}42\,U = U_2$

$U_t = U_2 \dfrac{R_4}{R_3+R_4} = \ldots = 39{,}6\ V$

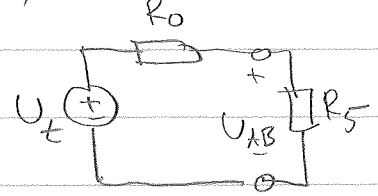
Ekv. Resistans. (nollställ U)

$R_0 = R_4 /\!/ (R_3 + R_1 /\!/ R_2) = \dfrac{R_4 \cdot \left(R_3 + \frac{R_1 R_2}{R_1+R_2}\right)}{R_4 + R_3 + \frac{R_1 R_2}{R_1+R_2}} = \ldots = 99\ \Omega$

Svar a/



b/

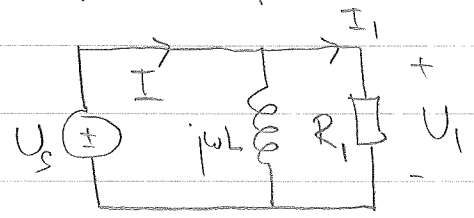


Sp. delning ger

$U_{AB} = U_t \cdot \dfrac{R_5}{R_0+R_5}$

$U_{AB} = 39{,}6 \cdot \dfrac{100}{99+100} = \underline{19{,}9\ V}$

3. jω-transformera



$i(t) = 0.165 \cos(\omega t)$ A

$\omega = 700$ r/s

$R_1 = 180\ \Omega$

$L = 215$ mH

$I = 0.165 \angle 0°$

Strömdelning:

$$I_1 = I \frac{j\omega L}{R_1 + j\omega L}$$

och

$$U_1 = I_1 R_1 = I \frac{j\omega L R_1}{R_1 + j\omega L} = I \frac{j\omega L}{1 + j\omega \frac{L}{R_1}} =$$

$$= I \frac{j\,700 \cdot 0{,}215}{1 + j\,\frac{700 \cdot 0{,}215}{180}} = I \cdot \frac{150{,}5 \angle 90°}{1{,}30 \angle 39{,}9°} =$$

$$= I \cdot 115{,}5 \angle 50{,}1° = 0{,}165 \cdot 115{,}5 = 19{,}1 \angle 50{,}1°$$

$u_1(t)$ efterfrågas så

$u_1(t) = 19{,}1 \cos(700t + 50{,}1°)$ V

4.

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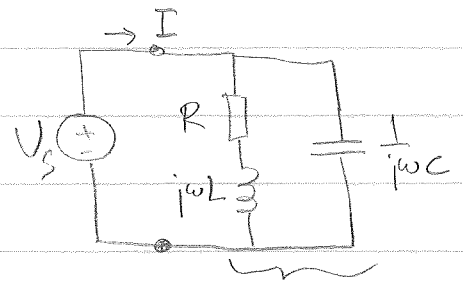
jω - transformera



$$Z = (R + j\omega L) \,/\!/\, \frac{1}{j\omega C}$$

$R = 100\ \Omega$

$L = 1{,}0\ H$

$C = 100\ \mu F$

$u_s(t) = 25\cos(\omega t)$

$U_s = 25\angle 0°$

$\omega = 200$

$$U_s = Z \cdot I$$

$$S = \frac{1}{2} U_s I^* = \frac{1}{2} U_s \left(\frac{U_s}{Z}\right)^* = \frac{1}{2} \frac{|U_s|^2}{Z^*} \cdot \frac{Z}{Z} = \frac{1}{2} \frac{|U_s|^2}{|Z|^2} \cdot Z$$

$$Z = \frac{(R + j\omega L)\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC} =$$

$$= \frac{100 + j200 \cdot 1}{1 - 200^2 \cdot 1 \cdot 100 \cdot 10^{-6} + j\, 200 \cdot 100 \cdot 100 \cdot 10^{-6}} = \frac{100(1+j2)}{-3 + j2} =$$

$$= \frac{100(1+j2)(3+j2)}{-(3-j2)(3+j2)} = \frac{-100(-1+j8)}{13} = \frac{100}{13}(1 - j8)$$

$$Z = 62{,}0 \angle -82{,}9°\ \Omega$$

P: Medeleffekt som avges av källa = Medeleffekt som upptas av Z.

$$P = \mathrm{Re}\{S\} = \frac{1}{2} \frac{|U_s|^2}{|Z|^2} \cdot \mathrm{Re}\{Z\} = \frac{1}{2} \cdot \frac{25^2}{(62{,}0)^2} \cdot \frac{100}{13}$$

$$P \approx 0{,}63\ W$$

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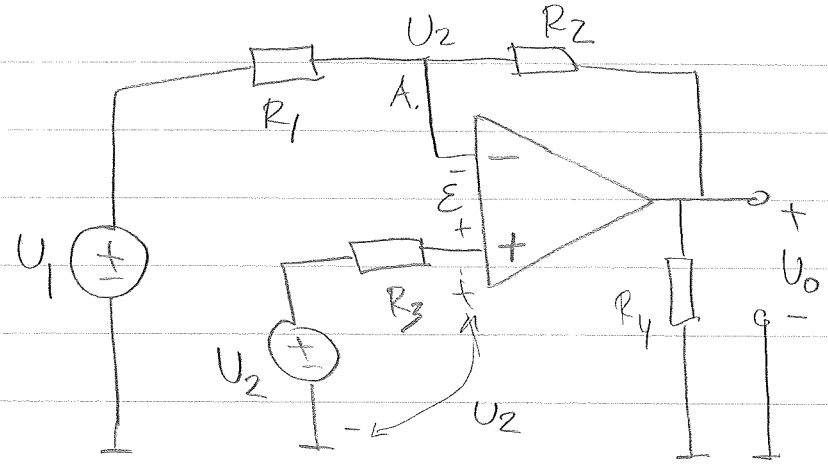
5.



$R_3 = 10\ k\Omega$

$R_1 = 2R_3 = 20\ k\Omega$

$R_2 = 3R_1 = 60\ k\Omega$

Neg återkoppling, Ideal op. först $\Rightarrow \varepsilon = 0,\ i_{op} = 0$

Spänning vid "+" och "−" ingång relativt jord $= U_2$.

$$KCL_A: \quad \frac{U_1 - U_2}{R_1} + \frac{U_0 - U_2}{R_2} = 0$$

$$\frac{U_0}{R_2} = \frac{U_2}{R_1} + \frac{U_2}{R_2} - \frac{U_1}{R_1} = U_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - U_1\frac{1}{R_1}$$

$$U_0 = U_2\left(1 + \frac{R_2}{R_1}\right) - U_1\frac{R_2}{R_1}$$

$$U_0 = U_2\left(1 + \frac{3R_1}{R_1}\right) - U_1 \cdot \frac{60}{20} = 4U_2 - 3U_1$$

Svar: $U_0 = 4U_2 - 3U_1$

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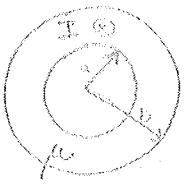
Ström I är likformigt fördelad inom ledare.

Elektrisk strömtäthet : $J = \frac{I}{S} = \frac{I}{\pi(b^2 - a^2)}$



Enligt Ampères lag $\oint \vec{B}\, d\vec{s} = \mu I_0$

$$B(r) \cdot 2\pi r = \mu I_0$$

$$B(r) = \frac{\mu I_0}{2\pi r}$$

a) $r \leq a$

$I_0 = 0 \quad \Rightarrow \quad B(r) = 0$

b) $a < r < b$

$$I_0 = J \cdot \pi(r^2 - a^2) = \frac{I}{\pi(b^2 - a^2)} \cdot (r^2 - a^2) = \frac{I(r^2 - a^2)}{b^2 - a^2}$$

$$B(r) = \frac{\mu I (r^2 - a^2)}{b^2 - a^2} \cdot \frac{1}{2\pi r}$$

c) $r \geq b$

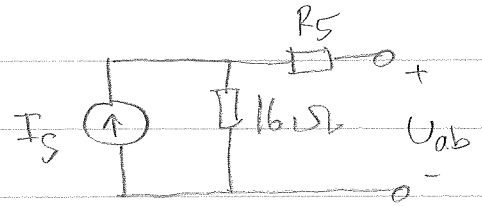
$I_0 = I \qquad B(r) = \frac{\mu_0 I}{2\pi r}$

$\mu = \mu_0$

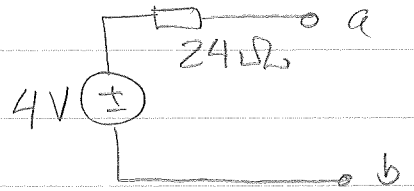
Svar: eem076 2013-01-15

1/ $U_{tot} = \ldots = \dfrac{-U}{1 + \frac{R_1}{R_3} \cdot \frac{1}{(1+k)}} = -4\ V$

2/ Först



$\Rightarrow$ Thevenin



3/ $Z = \dfrac{1}{j\omega C} /\!/ (R_2 + j\omega L)$

$I_S = \dfrac{U_S}{R_1 + Z} = \dfrac{20\angle 0^\circ}{150 + 50} = 0{,}1\angle 0^\circ\ A$

$\Rightarrow i_S(t) = 0{,}1 \cos(10t)\ A$

4/ $P_{R_1} = 7{,}2\ W$

5/ $U_0 = 3\ V$

6/ a) $B_y^{övre} = \dfrac{\mu_0 I}{2\pi y_1}$ $\quad y_1 = 1{,}0 + 0{,}004$

$B_y^{undre} = \dfrac{\mu_0 I}{2\pi y_2}$ $\quad y_2 = 1{,}0 - 0{,}004$

$B_y^{tot} = B_y^{övre} + B_y^{undre} = 3{,}2 \cdot 10^{-9} \sin(100\pi t)\ T$

c/ $B_y = \dfrac{\mu_0 I}{2\pi r_1} + \dfrac{\mu_0 I}{2\pi r_2} = \{r_1 = r_2 = 0{,}004\} = \ldots = 2{,}0 \cdot 10^{-4} \sin(100\pi t)\ T$