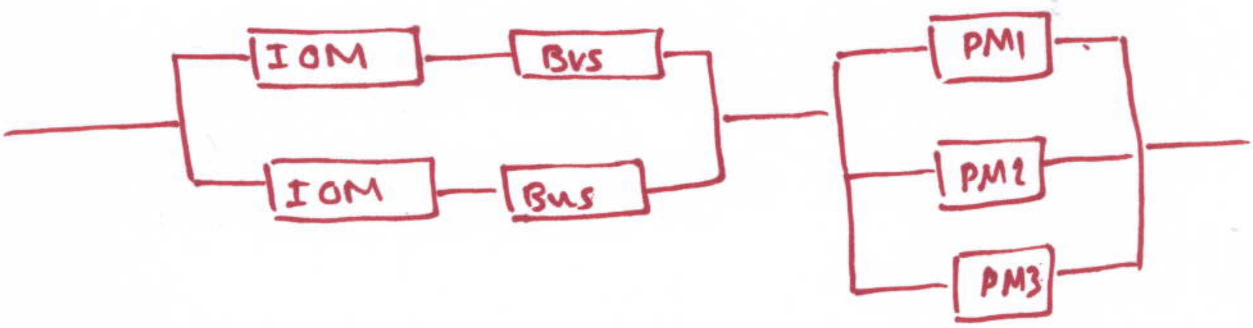
Problem 1.

1.a) 7 regions

1. PM1
2. PM2
3. PM3
4. (IOM – Parallel Bus)$_1$
5. (IOM – Parallel Bus)$_2$
6. Serial Bus 1
7. Serial Bus 2

1.b)

$$R_{system} = R_{IOM-Bus} * R_{PM}$$



$$R_{system} = (1 - (1 - R_{IOM-Bus})^2) *$$

$$(1 - (1 - R_{PM})^3)$$

$$(R_{IOM-Bus})_{1/2} = R_{IOM} * R_{Bus} = e^{-\lambda_2 t} * e^{-\lambda_3 t}$$

$$(R_{IOM-Bus})_{unit} = (1 - (1 - e^{(-\lambda_2 - \lambda_3)t})^2)$$

$$R_{PM} = e^{-\lambda_1 t}$$

$$R_{system}(t) = (1 - (1 - e^{(-\lambda_2 - \lambda_3)t})^2) * (1 - (1 - e^{-\lambda_1 t})^3)$$

Problem 1. cont.

1.c) $$MTTF_{IOM-Bus} = \int_0^\infty R_{IOM-Bus}(t)\, d(t)$$

$$= \int_0^\infty \left(1 - \left(1 - e^{(-\lambda_2 - \lambda_3)t}\right)^2\right) dt$$

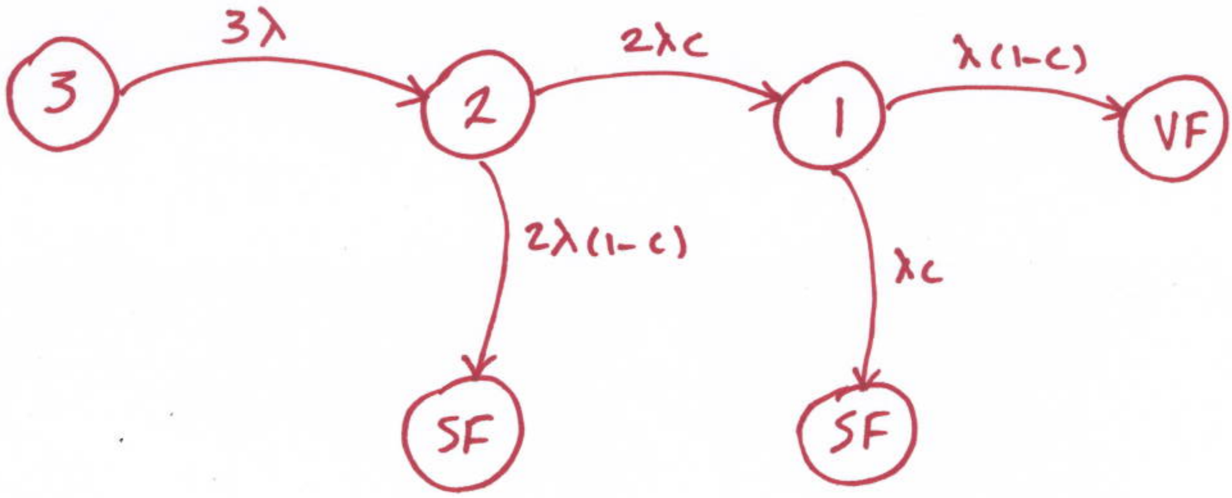
$$= \int_0^\infty \left(2e^{-(\lambda_2+\lambda_3)t} - e^{-2(\lambda_2+\lambda_3)t}\right) dt$$

$$= \frac{-2}{\lambda_2+\lambda_3} e^{-(\lambda_2+\lambda_3)t} + \frac{1}{2(\lambda_2+\lambda_3)} e^{-2(\lambda_2+\lambda_3)t} \Big]_0^\infty$$

$$= \left(0 + \frac{2}{\lambda_2+\lambda_3}\right) + \left(0 - \frac{1}{2(\lambda_2+\lambda_3}\right)$$

$$= \frac{3}{2} \cdot \frac{1}{\lambda_2+\lambda_3}$$

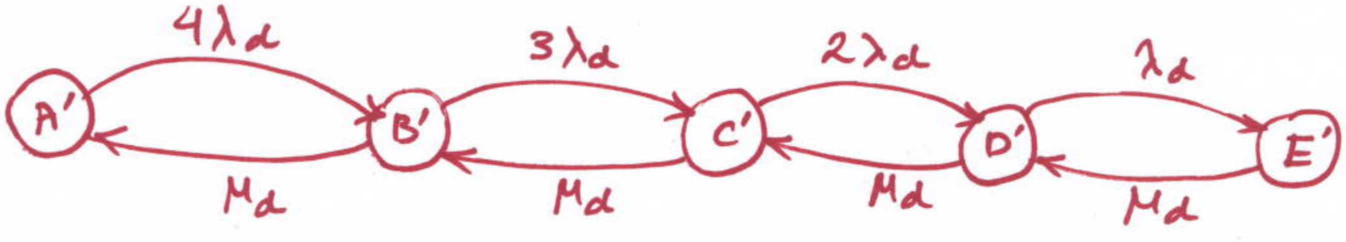
Problem 7.d)



steady-state probability that the processor modules subsystem cause the node to exhibit a silent failure = $1 - c + c^2$

Problem 2

Markov Model for Disks



This is a Birth & Death process.

$$\pi_{B'} = \frac{4\lambda_d}{\mu_d}\pi_{A'}$$

$$\pi_{C'} = \frac{3\lambda_d}{\mu_d}\pi_{B'} = \frac{3\lambda_d}{\mu_d}\cdot\frac{4\lambda_d}{\mu_d}\pi_{A'} = \frac{12\lambda_d^2}{\mu_d^2}\pi_{A'}$$

$$\pi_{D'} = \frac{2\lambda_d}{\mu_d}\pi_{C'} = \frac{2\lambda_d}{\mu_d}\cdot\frac{12\lambda_d^2}{\mu_d^2}\pi_{A'} = \frac{24\lambda_d^3}{\mu_d^3}\pi_{A'}$$

$$\pi_{E'} = \frac{\lambda_d}{\mu_d}\pi_{D'} = \frac{\lambda_d}{\mu_d}\cdot\frac{24\lambda_d^3}{\mu_d^3}\pi_{A'} = \frac{24\lambda_d^4}{\mu_d^4}\pi_{A'}$$

$$\pi_{A'} + \pi_{B'} + \pi_{C'} + \pi_{D'} + \pi_{E'} = 1$$

$$\pi_{A'}\left(1 + \frac{4\lambda_d}{\mu_d} + \frac{12\lambda_d^2}{\mu_d^2} + \frac{24\lambda_d^3}{\mu_d^3} + \frac{24\lambda_d^4}{\mu_d^4}\right) = 1$$

$$\pi_{A'} = \frac{\mu_d^4}{\mu_d^4 + 4\lambda_d\mu_d^3 + 12\lambda_d^2\mu_d^2 + 24\lambda_d^3\mu_d + 24\lambda_d^4}$$

$$\pi_{Disks} = \pi_{A'} + \pi_{B'} + \pi_{C'} + \pi_{D'}$$

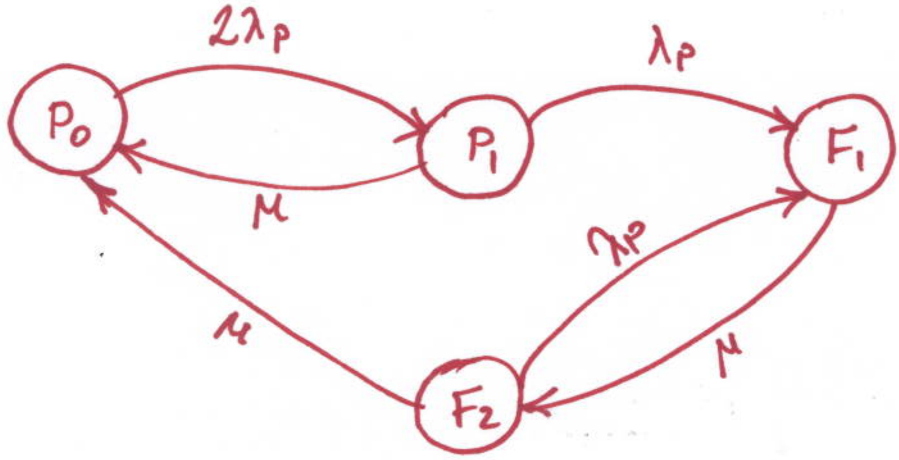
Problem 2. cont.

$$\Pi_{Disks} = \Pi_{A'} \left(1 + \frac{4\lambda_d}{\mu_d} + \frac{12\lambda_d^2}{\mu_d^2} + \frac{24\lambda_d^3}{\mu_d^3} \right)$$

$$\Pi_{Disks} = \frac{\mu_d^4 + 4\lambda_d\mu_d^3 + 12\lambda_d^2\mu_d^2 + 24\lambda_d^3\mu_d}{\mu_d^4 + 4\lambda_d\mu_d^3 + 12\lambda_d^2\mu_d^2 + 24\lambda_d^3\mu_d + 24\lambda_d^4}$$

Problem 2. cont.

Markov Model for Processors



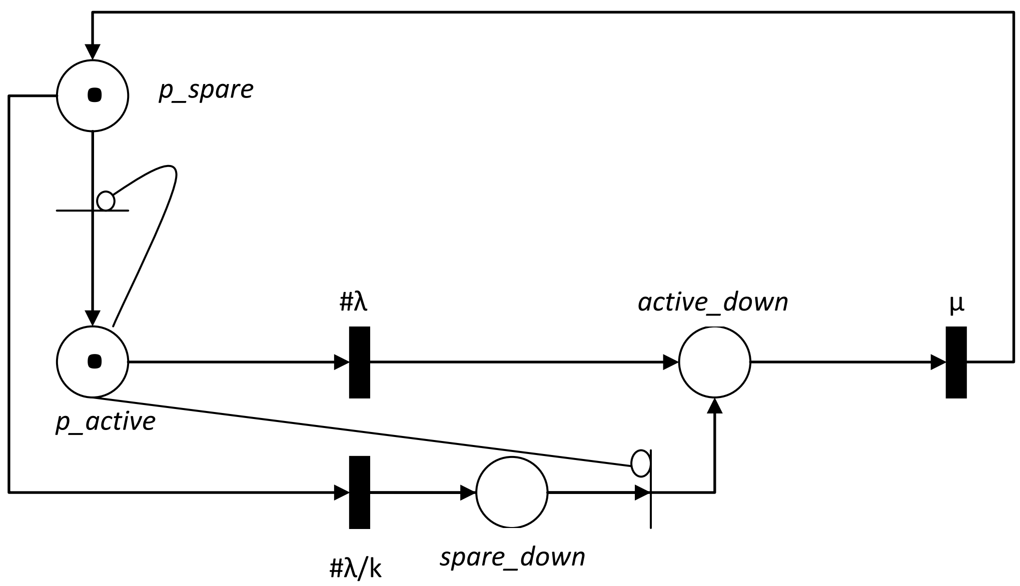
$$[0 \quad 0 \quad 0 \quad 0] = [\Pi_{P_0} \quad \Pi_{P_1} \quad \Pi_{F_1} \quad \Pi_{F_2}] \cdot Q$$

$$Q = \begin{bmatrix} -2\lambda_p & 2\lambda_p & 0 & 0 \\ \mu & -\lambda_p - \mu & \lambda_p & 0 \\ 0 & 0 & -\mu & \mu \\ \mu & 0 & \lambda_p & -\mu - \lambda_p \end{bmatrix}$$

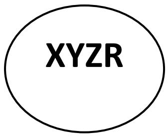
$$\Pi_{processors} = \Pi_{P_0} + \Pi_{P_1}$$

$$\Pi_{system} = \Pi_{processors} * \Pi_{disks}$$

3.a



3.b



X: # of tokens in *p_spare*

Y: # of tokens in *p_active*

Z: # of tokens in *active_down*

R: # of tokens in *spare_down*